Beyond Two Dark Energy Parameters

Devdeep Sarkar

in collaboration with:
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Daniel Holz (Chicago/LANL), and Asantha Cooray (UCI)

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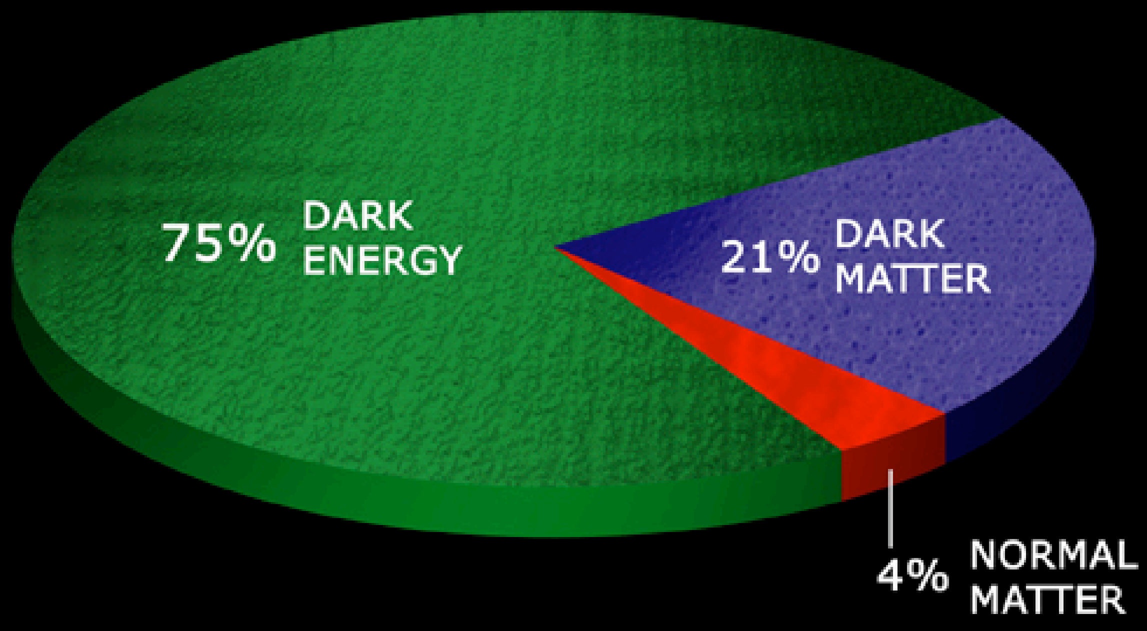
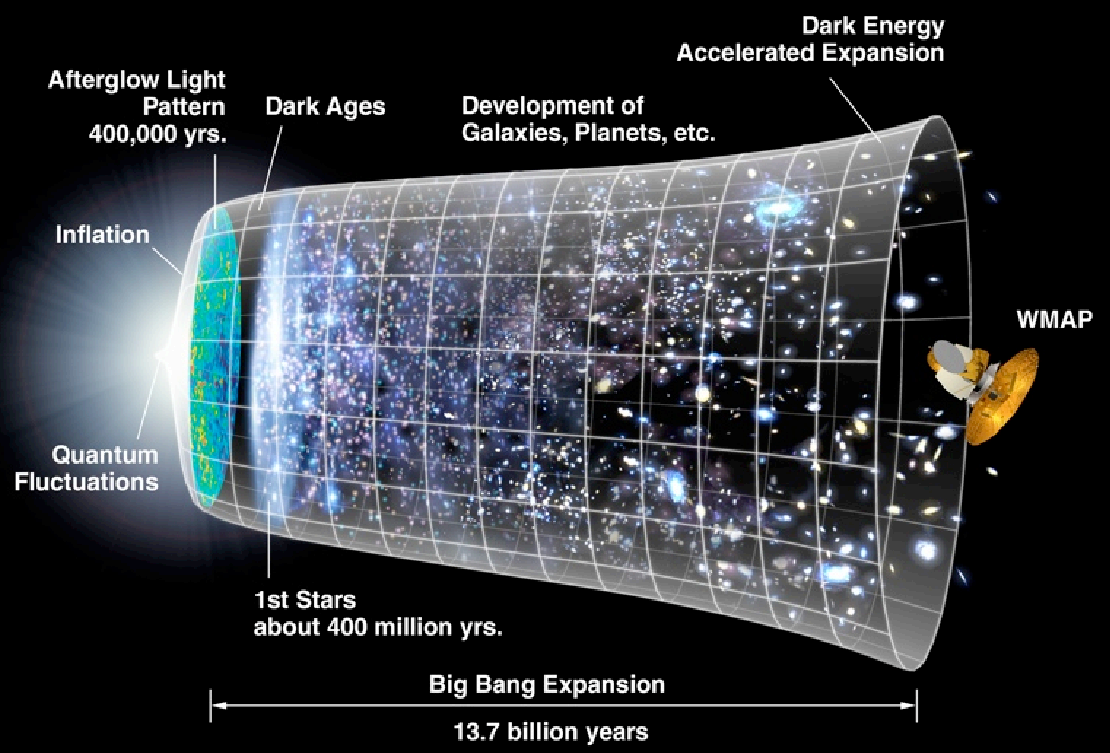
Energy Budget of the Universe

$$\Omega_{Total} = \Omega_b + \Omega_{dm} + \Omega_{de} \simeq 1$$

$$\left(\Omega_X \equiv \frac{\rho_X}{\rho_{crit}} \right)$$





Puzzle! Puzzle! Puzzle!

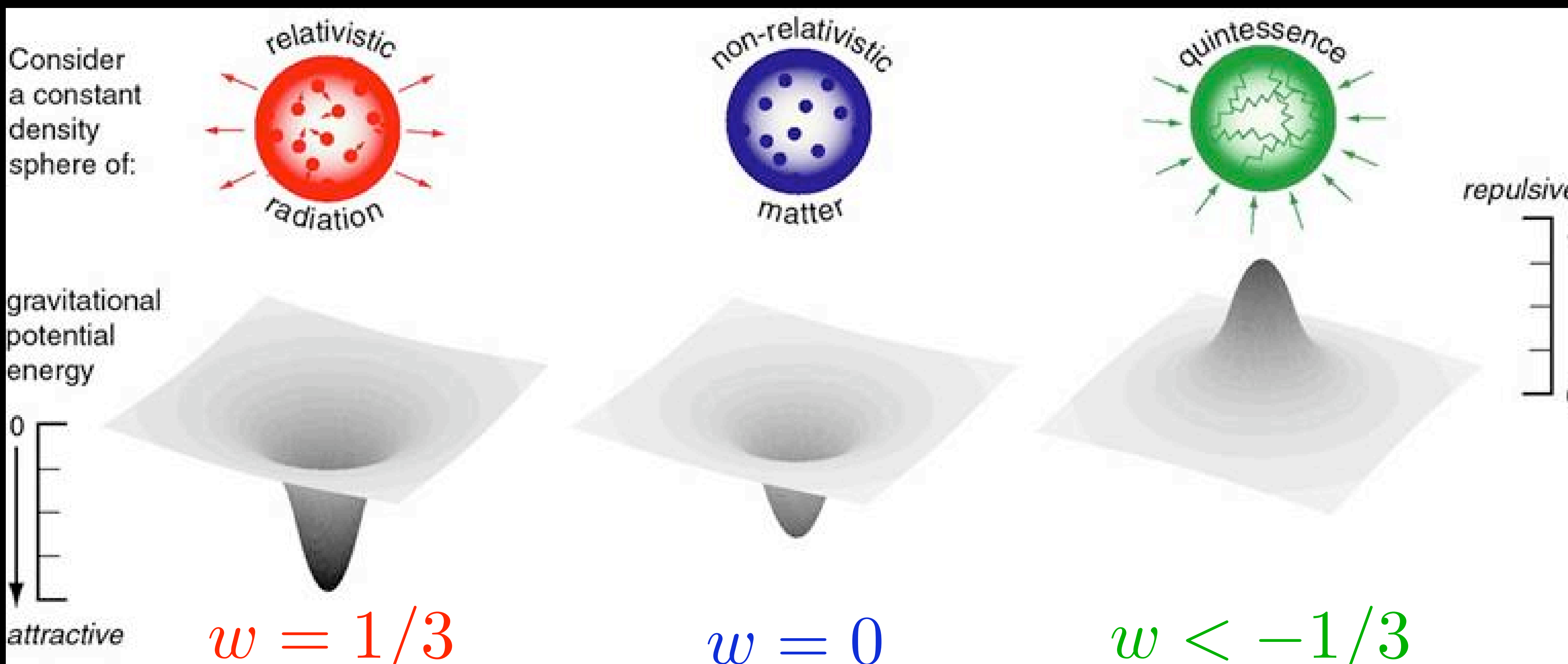
What is Dark Energy?

"Dark Energy is made from an exclusive blend of vital L-amino acids, beneficial vitamins and bionutrients that allows faster and greater ion penetration of the cell walls, visibly enhancing the rate of growth"

GrowLightSource.com

Dark Energy Equation Of State

$$T_\mu^\nu = diag(\rho, -p, -p, -p) \qquad p = w\rho$$



For Cosmological Constant... $w = -1$

“Seeing” The Dark Energy

"Seeing" The Dark Energy

...via its effect on the expansion of the Universe

$$H(z) = H_0 \left[\Omega_m (1+z)^3 + \Omega_k (1+z)^2 + (1 - \Omega_k - \Omega_m) F(z)\right]^{1/2}$$

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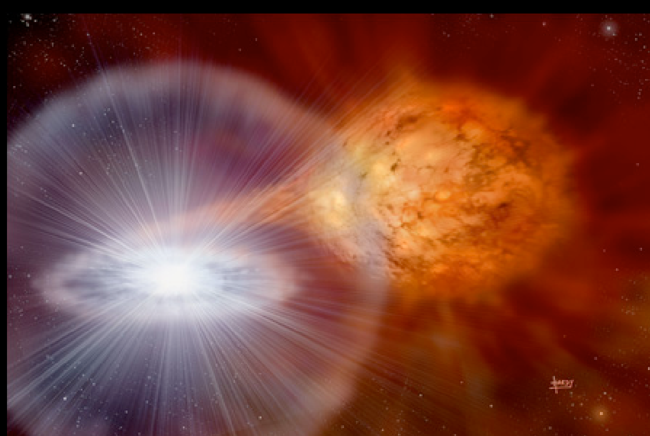

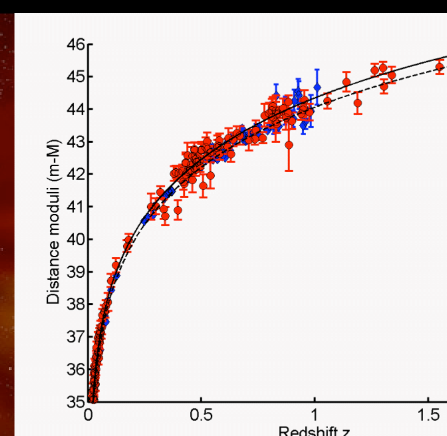

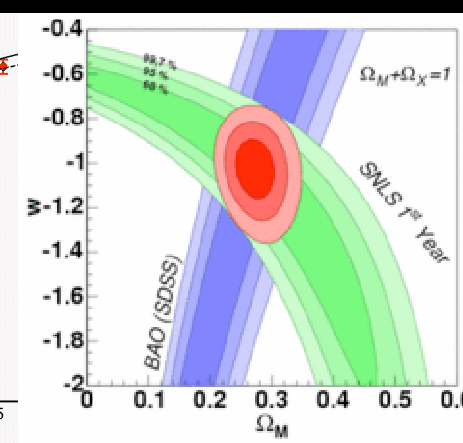

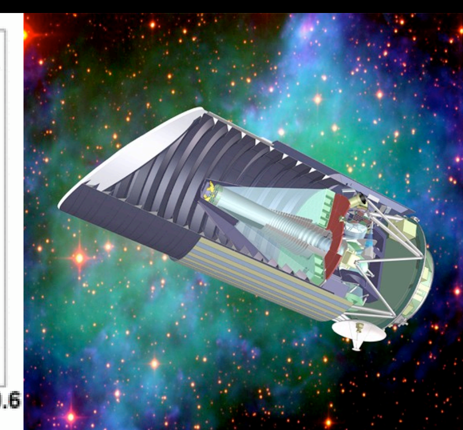

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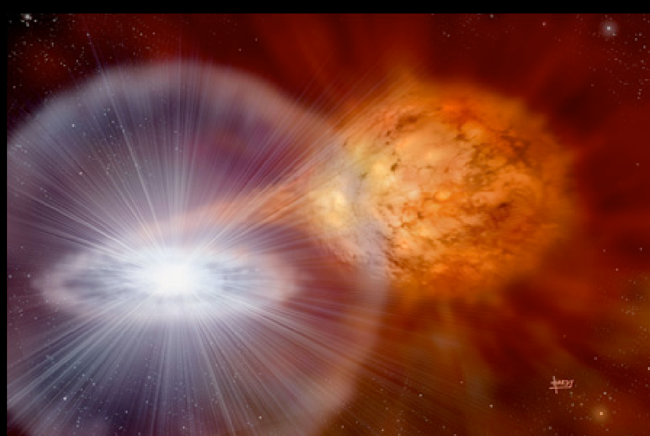
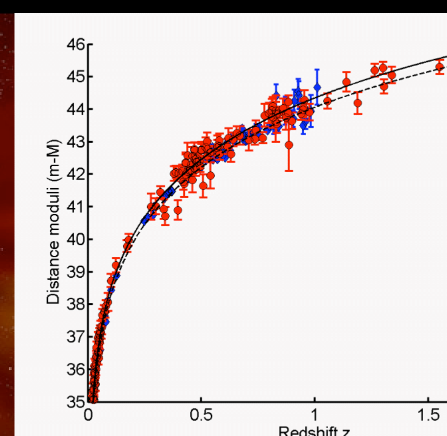
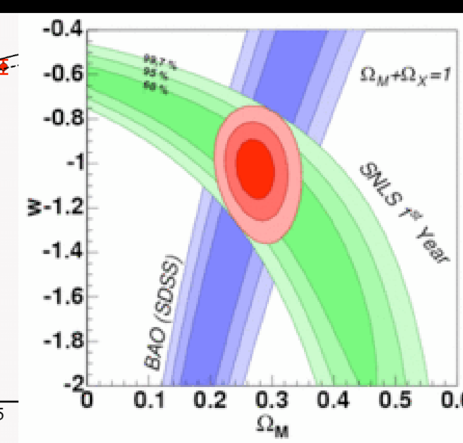
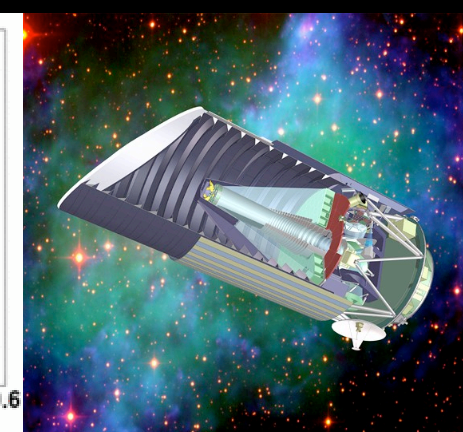
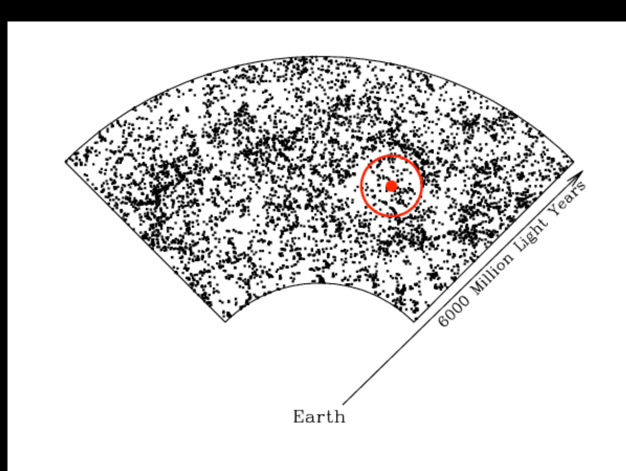


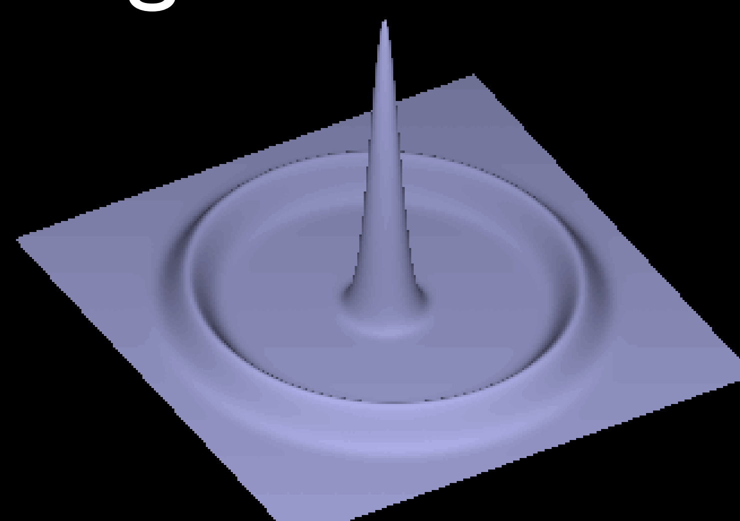


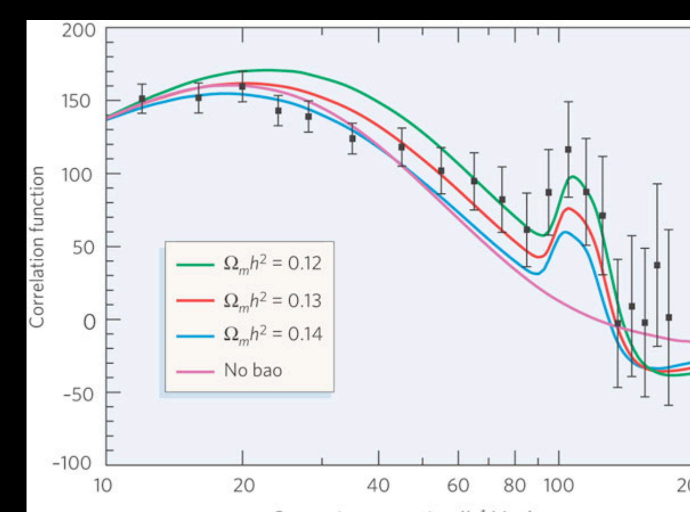




(2) Standard Rulers: Angular Diameter Distance via BAO







DE EOS Revisited: Different Approaches...

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(A) Parameterize w(z) [Adopted by the DETF]

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(Linder 2003)

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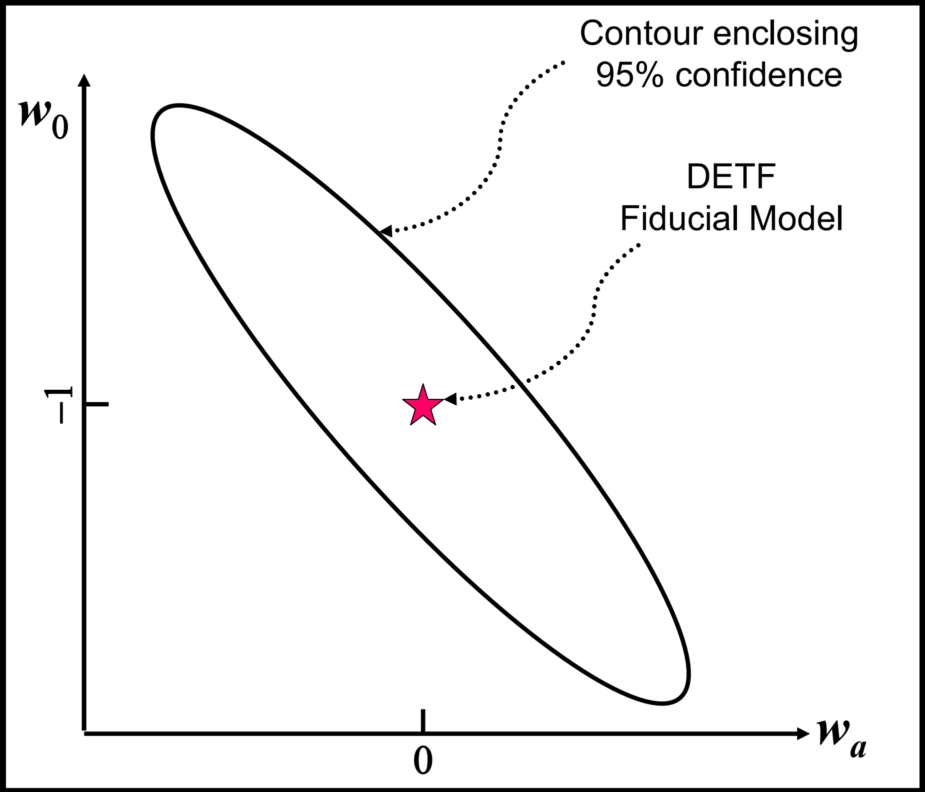
DETF Figure of Merit (FoM)

Reciprocal of the Area of the Error Ellipse enclosing the 95% Confidence Limit in the $w_0 - w_a$ Plane.



Larger Value of the FoM ⟺ Greater Accuracy

DE EOS Revisited: Beyond Two Param...

(B) Model-Independent Redshift-Binning of w(z)

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bin	$w_1(z)$	$w_2(z)$	$w_3(z)$	$w_4(z)$	$w_5(z)$	$w_6(z)$
z-ran	0-0.07	0.07-0.15	0.15-0.3	0.3-0.6	0.6-1.2	1.2-2.0

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$$F(z_n > z > z_{n-1}) = (1+z)^{3(1+w_n)} \prod_{i=0}^{n-1} (1+z_i)^{3(w_i - w_{i+1})}$$

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Covariance Matrix

$$C = \left\langle \mathbf{w}\mathbf{w}^T \right\rangle - \left\langle \mathbf{w} \right\rangle \left\langle \mathbf{w}^T \right\rangle$$

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(C) Decorrelated Estimates of w(z)

(Huterer & Cooray 2005)

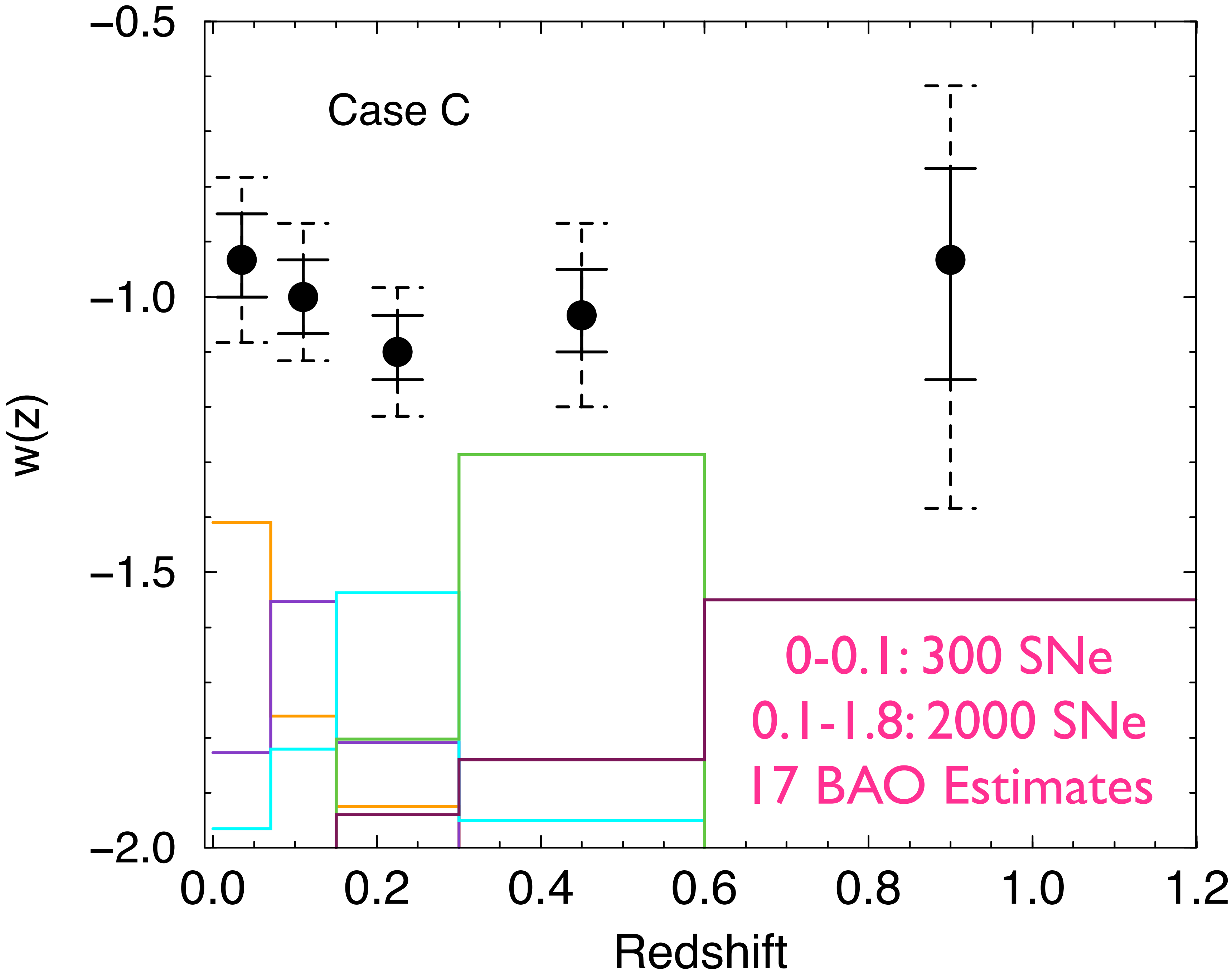
Diagonalize The Fisher Matrix

$$\mathbf{F} \equiv \mathbf{C}^{-1} = \mathbf{O}^T \mathbf{\Lambda} \mathbf{O}$$

Our Analyses: Six Mock Scenarios

- Case A: 200 SNe up to z=1.8; 2 BAO estimates
- Case B: 300 SNe out to z=0.1 & 2000 SNe in the range 0.1<z<1.8; 7 BAO estimates
- Case C: Same as (B)... with 10 new BAO constraints
- Case D: 10,000 SNe out to z=2.0, 7 BAO estimates
- Case E: 10,000 SNe out to z=2.0; 17 BAO estimates
- Case F: 200 SNe as in (A); 17 BAO estimates

Results



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D.S., S. Sullivan, S. Joudaki, A. Amblard, D. Holz, A. Cooray (2007)

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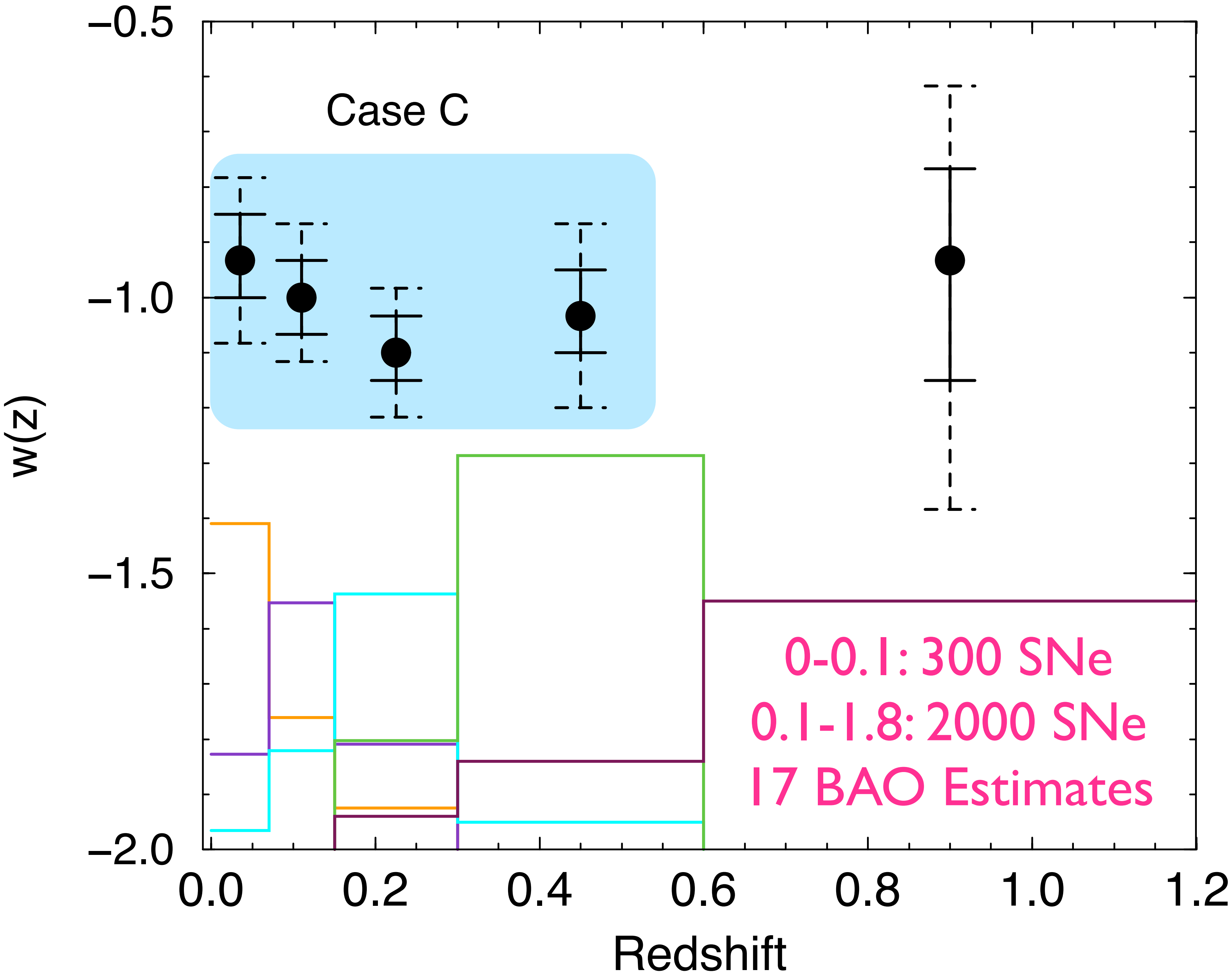

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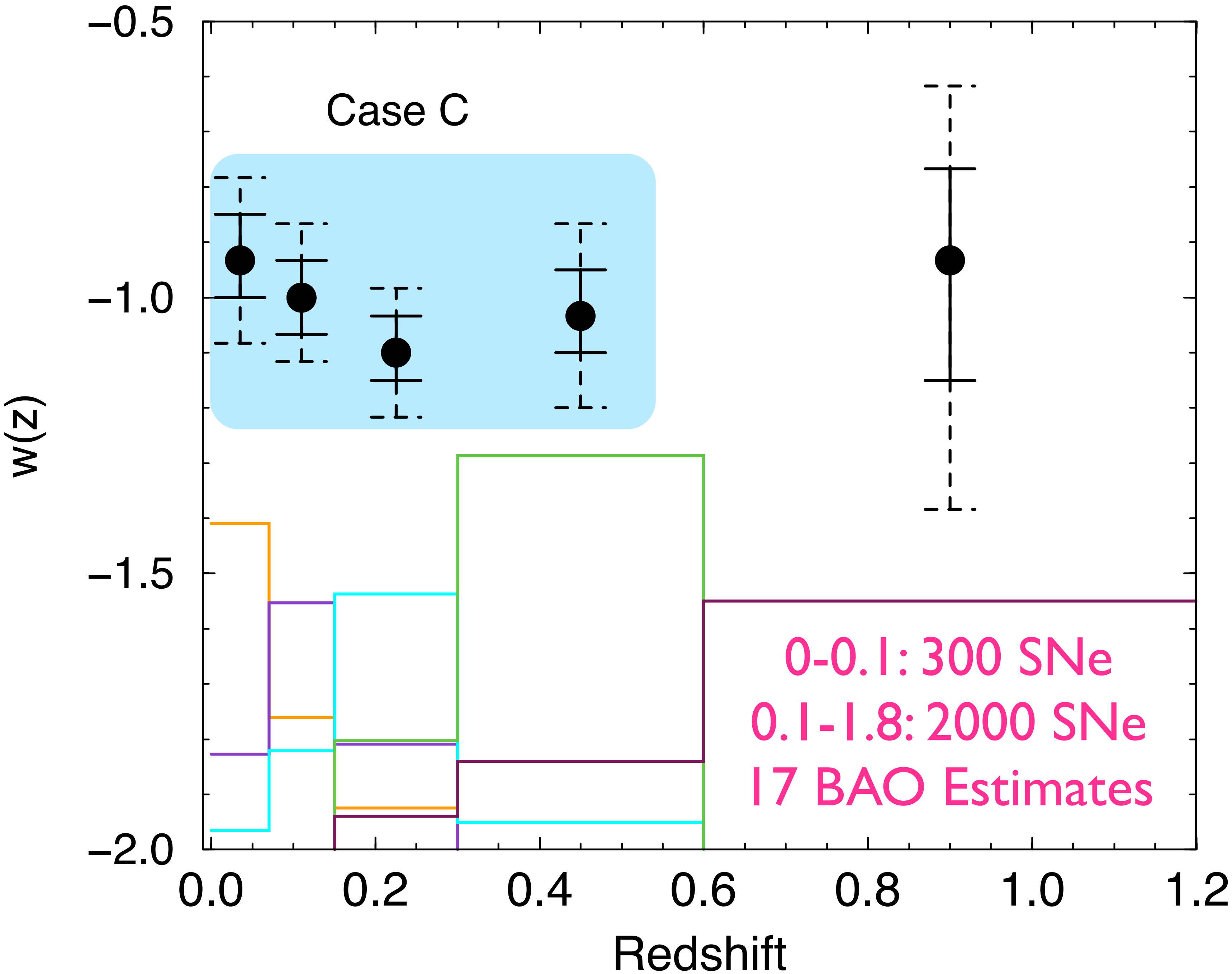
$$\mathrm{FoM} = \left[\sum_i \frac{1}{\sigma^2(w(z_i))}\right]^{1/2}$$



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Conclusion

- Future surveys can be expected to estimate three or more independent dark energy EOS parameters to an accuracy better than 10%
- We propose a parameterization-independent Figure of Merit (FoM):

$$\mathrm{FoM} = \left[\sum_i \frac{1}{\sigma^2(w(z_i))}\right]^{1/2}$$

- The FoM holds for an arbitrary # of dof


.com
THANK YOU
WELLS FARGO
We're on Track
For a Cure

Extra Slides

Constraining with Standard Candles (SNe)

$$m - M = -2.5\,\log_{10}\left[\frac{\mathcal{L}/4\pi d_L^2}{\mathcal{L}/4\pi(10\mathrm{pc})^2}\right]$$

$$= 5log_{10}\left(\frac{d_L}{\mathrm{Mc}}\right) + 25$$

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$$d_L(z) = (1+z)\frac{c}{H_0} \times \begin{cases} \frac{1}{\sqrt{|\Omega_k|}} \sinh\left(\sqrt{|\Omega_k|}\int_0^z \frac{dz'}{H(z')/H_0}\right) & \text{if } \Omega_k > 0, \\ \int_0^z \frac{dz'}{H(z')/H_0} & \text{if } \Omega_k = 0, \\ \frac{1}{\sqrt{|\Omega_k|}} \sin\left(\sqrt{|\Omega_k|}\int_0^z \frac{dz'}{H(z')/H_0}\right) & \text{if } \Omega_k < 0, \end{cases}$$

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$w_0 = -1$ and $w_1 = 0$ $\Longleftrightarrow$ Cosmological Constant

Different Scales

Redshift, z	Time, t (Years)	cz (km/s)	Distance (Mpc/h)	Temperature (K)
0	13.7 billion	0	0	2.725
0.01	13.5 billion	3000	30	2.752
0.033	13.1 billion	10,000	100	2.815
0.1	11.9 billion	30,000	300	2.998
1.0	04.8 billion	300,000	3000	5.450
4.0	~1 billion	1.2 million	12,000	13.625
1100	400,000	0.3 billion	3.3 million	3000

$$1 \text{ Mpc} = 10^6 \text{ pc} = 3.26 \times 10^6 \text{ ly} = 3.08 \times 10^{22} \text{ m}$$