

Cosmic Shear from Scalar-Induced Gravitational Wave

Devdeep Sarkar
Center for Cosmology, UC Irvine

Based on:

D.S., Paolo Serra, Asantha Cooray (UCI), Kiyotomo Ichiki (Tokyo), and Daniel Baumann (Princeton),
Phys. Rev. D, 77, 103515 (2008) [arXiv: 0803.1490]

UC Irvine

Astro Grad Seminar

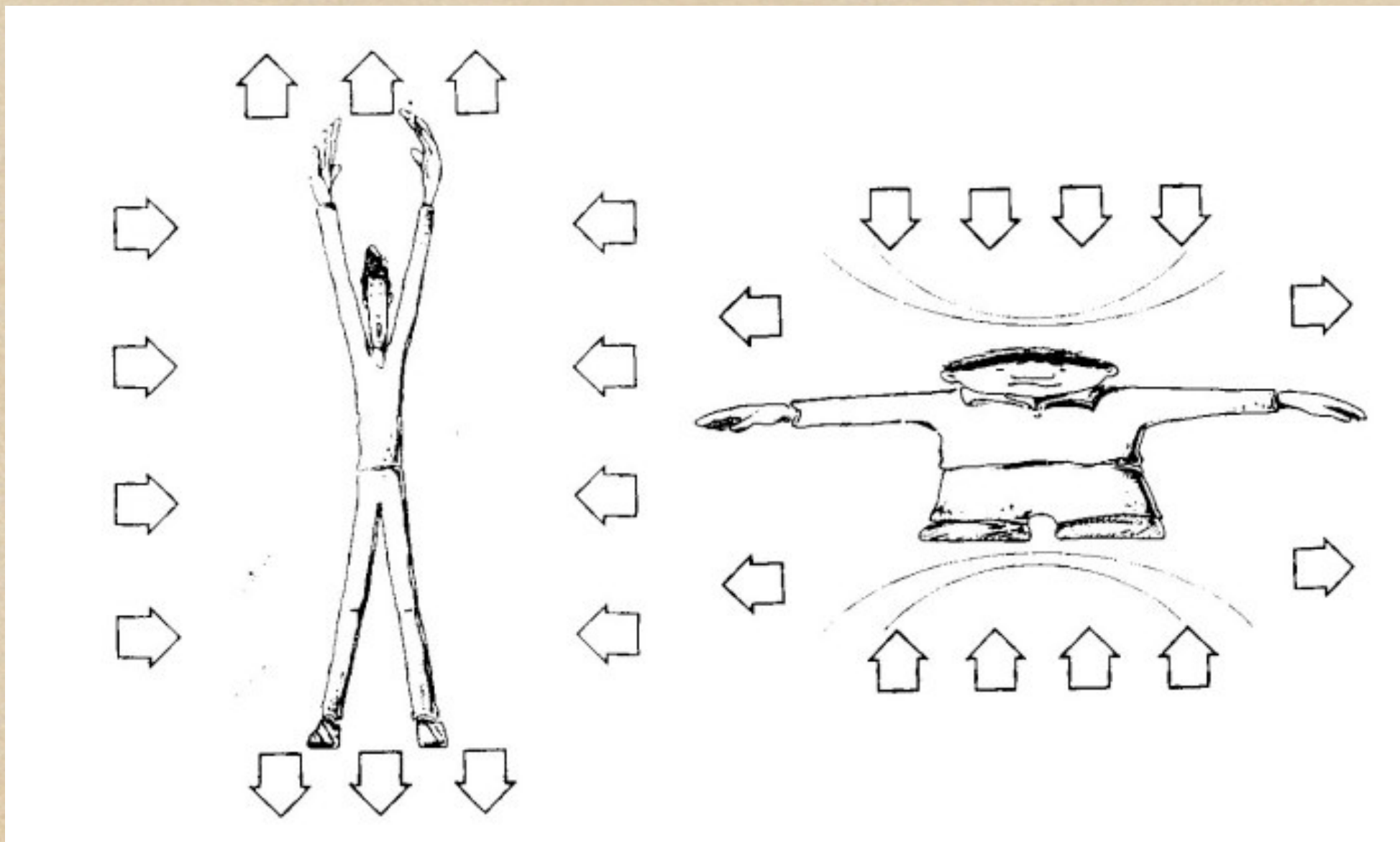
Oct 08, 2008

Agenda

- ABC's of Gravitational Waves
- Primary GWB vs Scalar-Induced GW
- Why Cosmic Shear?
- Gravitational Lensing 201
- Method and Results
- Conclusion

Motivation

To have a deep understanding of...



credit: <http://www.lnl.infn.it/~auriga/>

ABC's of Gravitational Waves

Einstein's Field Equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$$

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Second order differential equations for metric tensor

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} ; \quad |h_{\mu\nu}| \ll 1$$

Flat Background
diag(-1,+1,+1,+1)

Small Perturbation

trace reversed

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$$

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Under Lorentz (or Einstein or Hilbert or Fock) Gauge:

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Linearized Einstein Equation becomes:

$$\square \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu}$$

$$(\square \equiv -\partial_t^2 + \partial_x^2 + \partial_y^2 + \partial_z^2)$$

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In Vacuum...

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Weak-Field limit for a Stationary Spherical Source:

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$$h_{00} = -2\Phi$$

$$h_{i0} = 0$$

$$h_{ij} = -2\Phi\delta_{ij}$$

ABC's of Gravitational Waves

Weak-Field limit for a Stationary Spherical Source:

$$h_{00} = -2\Phi$$

$$h_{i0} = 0$$

$$h_{ij} = -2\Phi\delta_{ij}$$

And hence... the metric becomes:

$$ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Phi)(dx^2 + dy^2 + dz^2)$$

ABC's of Gravitational Waves

Consider the solution:

$$\bar{h}_{\mu\nu} = C_{\mu\nu} e^{ik_\sigma x^\sigma}$$

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10 independent
components

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$$\square \bar{h}_{\mu\nu} = 0 \Rightarrow k_\sigma k^\sigma = 0 \Rightarrow \omega^2 = \delta_{ij} k^i k^j$$

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(the wave vector is orthogonal to $C^{\mu\nu}$)

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4 equations
6 components
remain

ABC's of Gravitational Waves

The solution:

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Get rid of coordinate freedom ($x^\mu \rightarrow x^\mu + \zeta^\mu$)...

4 more constraints...
leaves 2 components

These two numbers represent the physical information characterizing our plane wave in this gauge.

ABC's of Gravitational Waves

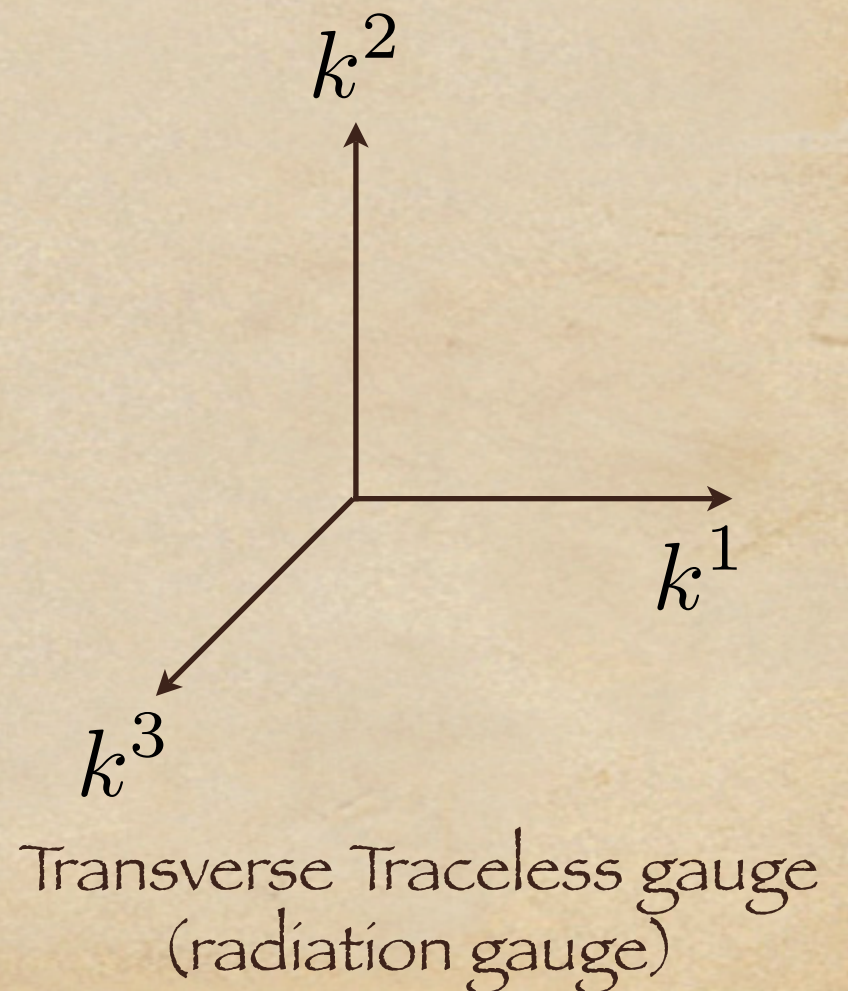
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Explicit construction...

$$k^\mu = (\omega, 0, 0, k^3) = (\omega, 0, 0, \omega)$$

$$C_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & C_{11} & C_{12} & 0 \\ 0 & C_{12} & -C_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



ABC's of Gravitational Waves

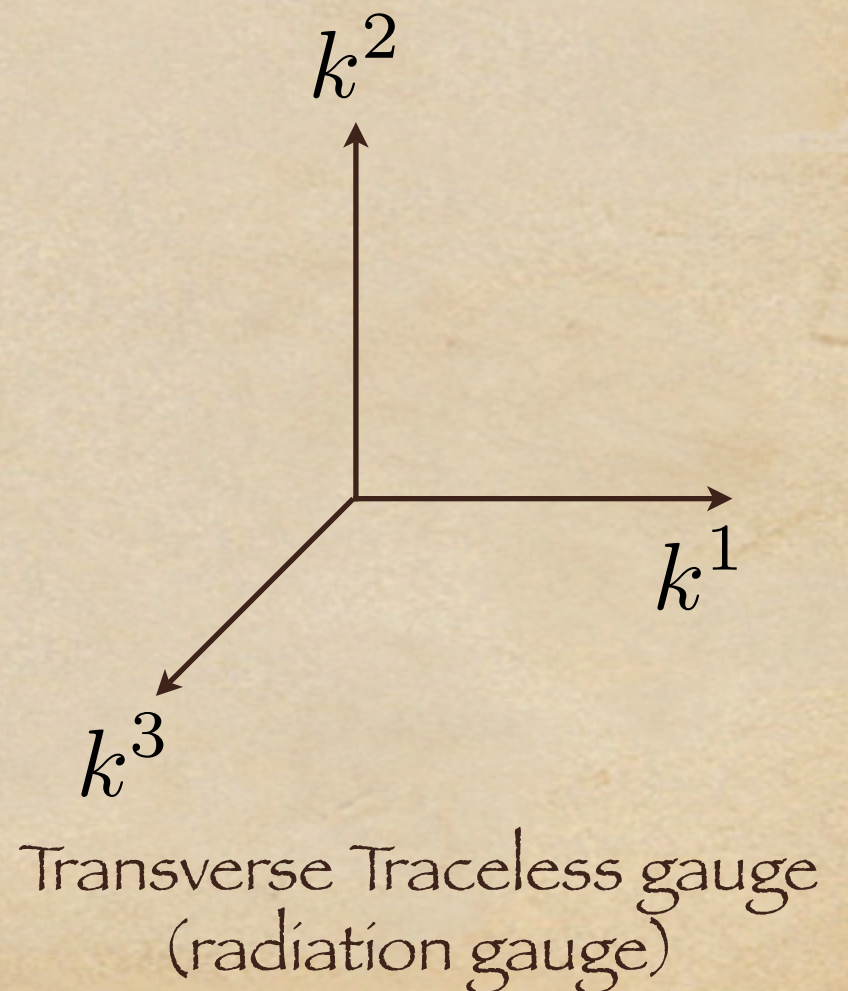
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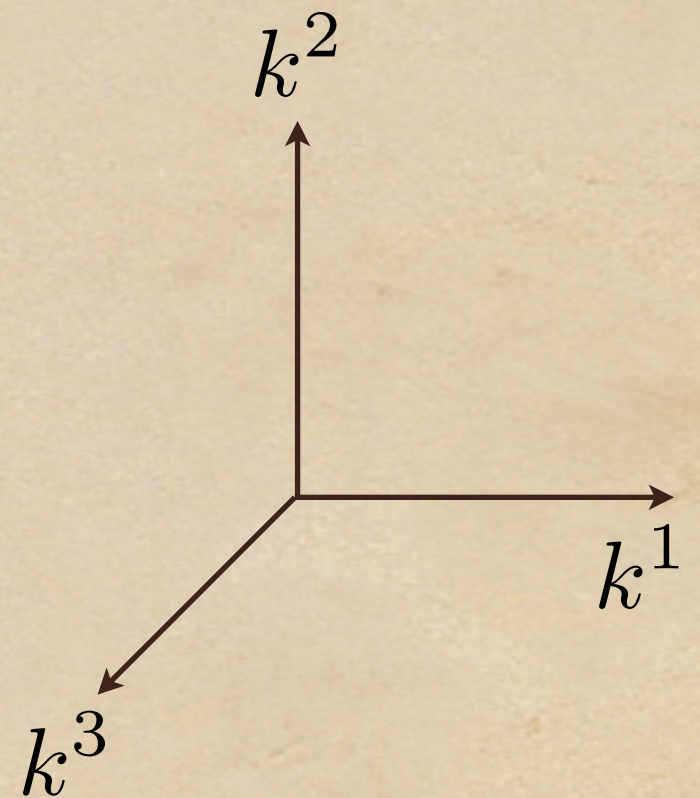


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Consider particles with separation S^μ



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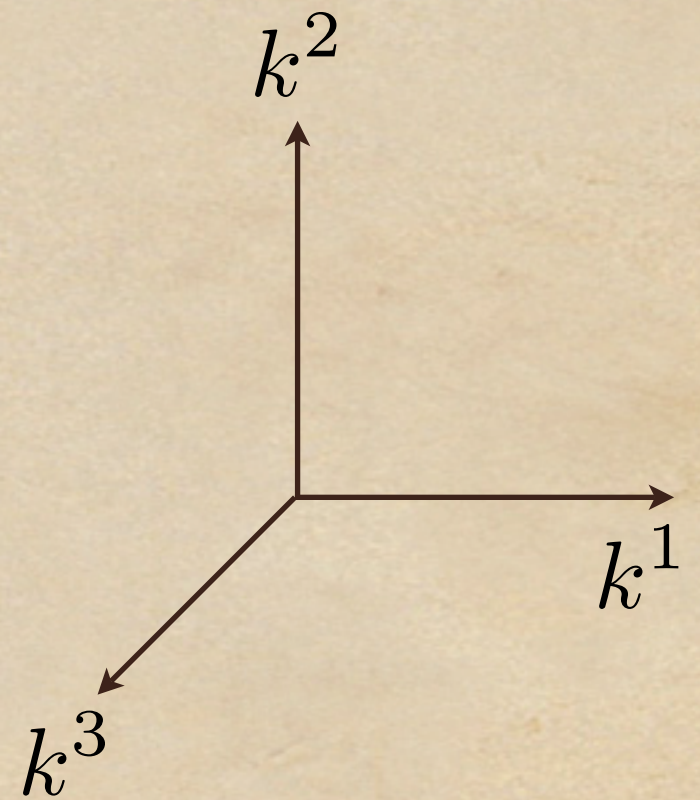
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Consider particles with separation S^μ

The geodesic deviation Equation:

$$\frac{\partial^2}{\partial t^2} S^\mu = \frac{1}{2} S^\sigma \frac{\partial^2}{\partial t^2} h^\mu_\sigma$$

This implies that ONLY S^1 and S^2
will be affected!



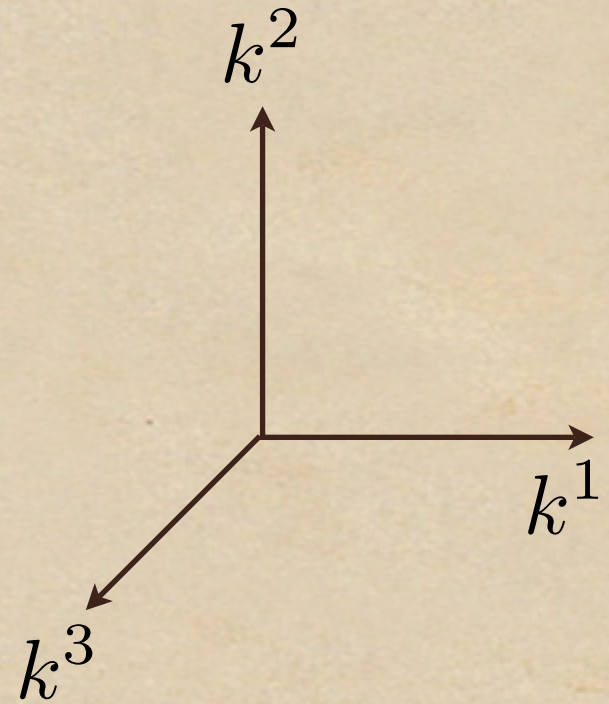
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ABC's of Gravitational Waves

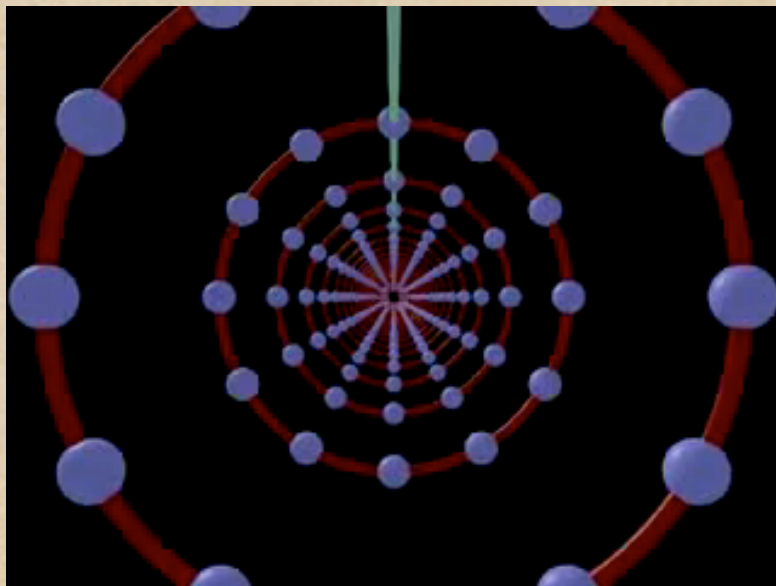
Case (I) $C_{\times} = 0$

$$S^1 = \left(1 + \frac{1}{2} C_+ e^{ik_{\sigma} x^{\sigma}} \right) S^1(0)$$

$$S^2 = \left(1 - \frac{1}{2} C_+ e^{ik_{\sigma} x^{\sigma}} \right) S^2(0)$$



Credit: Michael
Penn State Schuyllkill

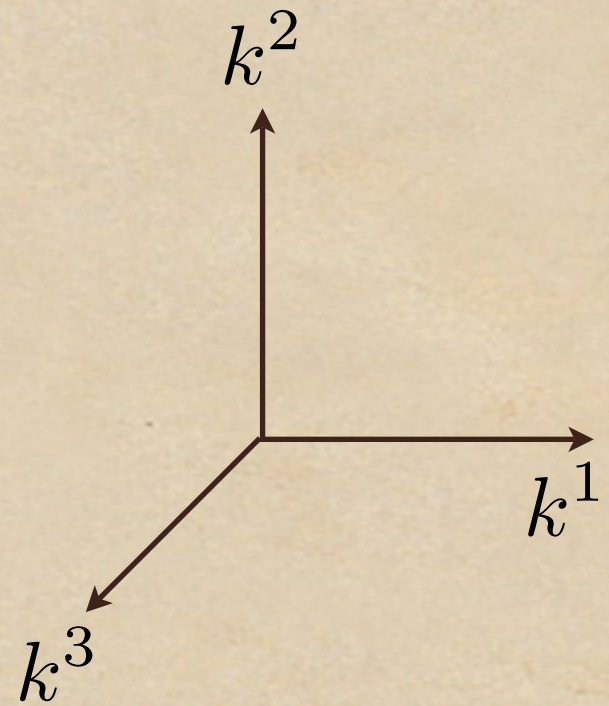


ABC's of Gravitational Waves

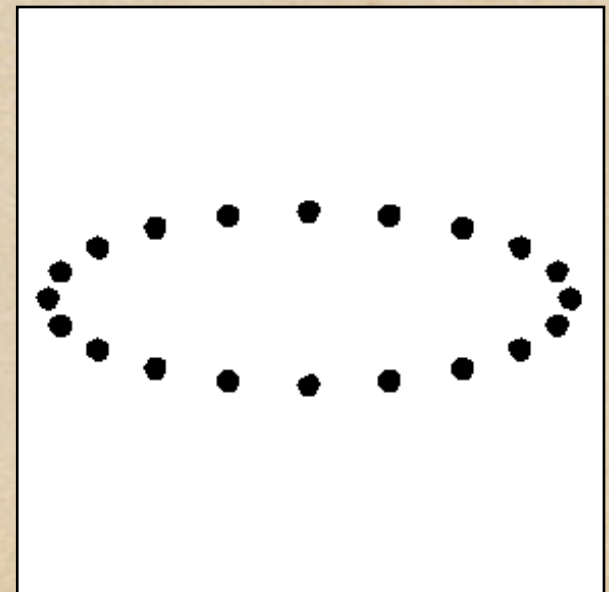
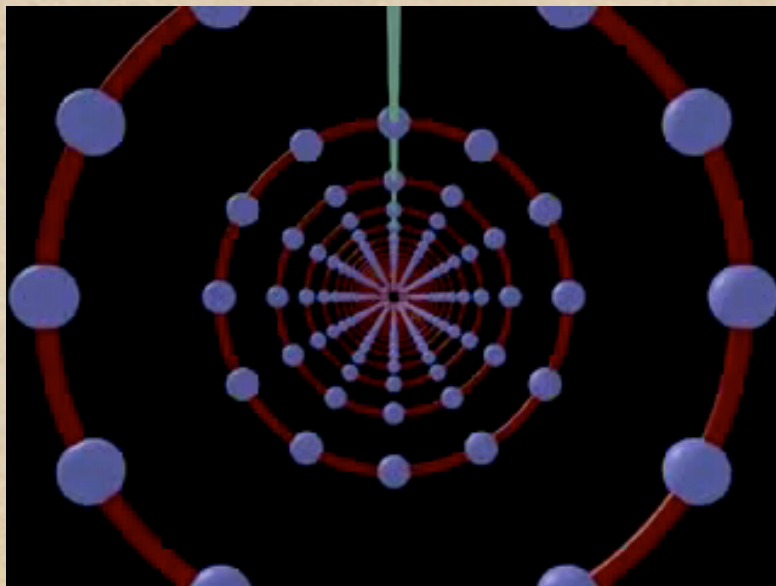
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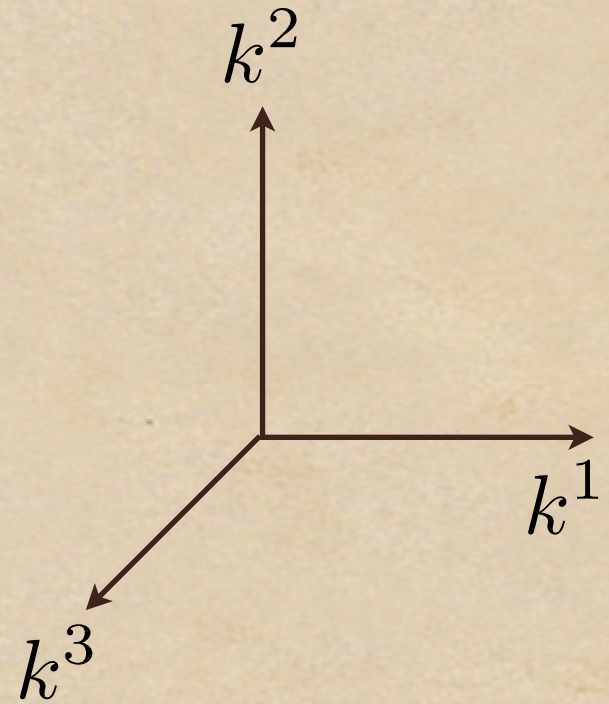


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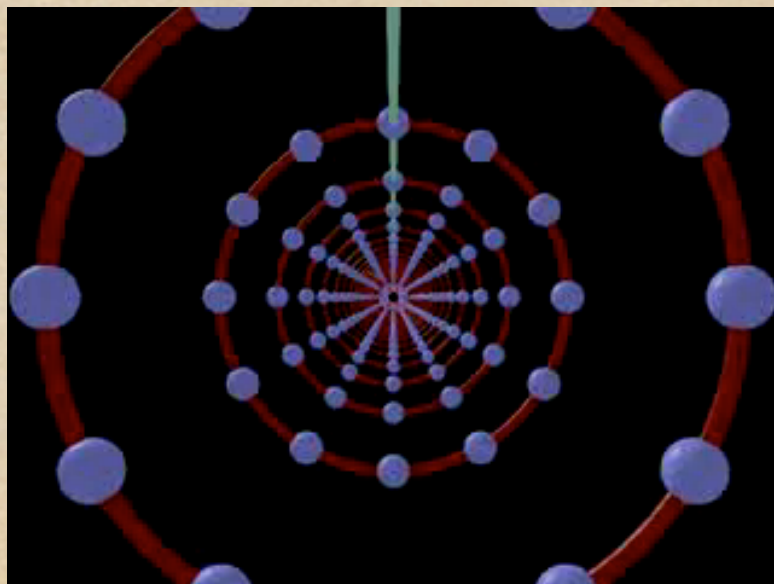
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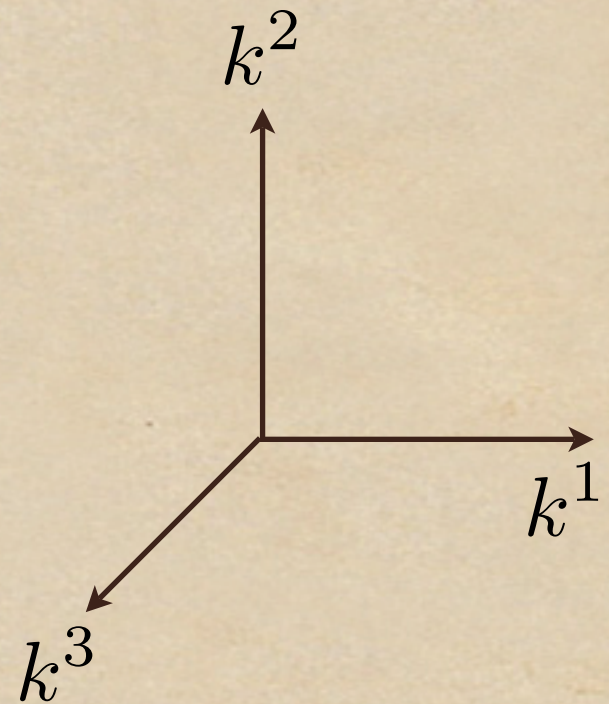


ABC's of Gravitational Waves

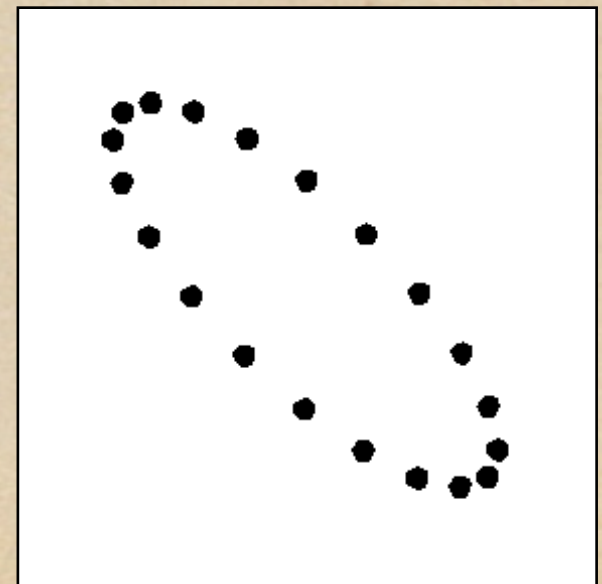
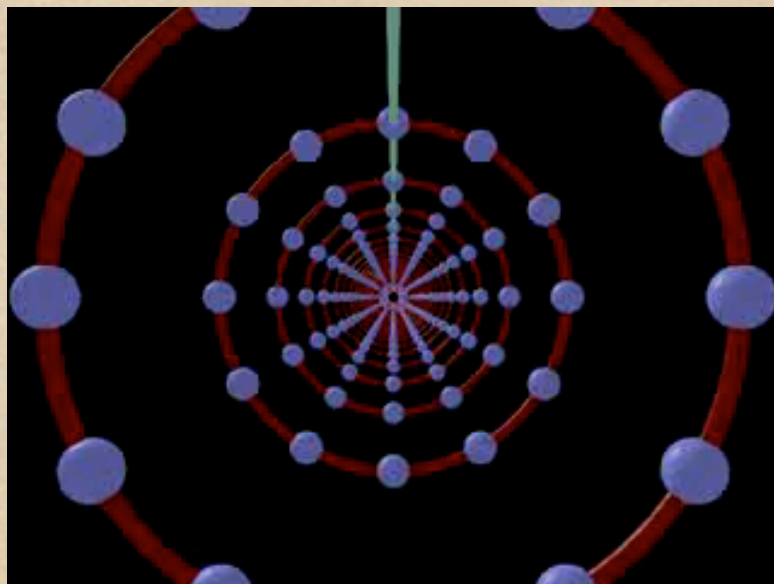
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Astrophysical Sources of GW



Artist's concept depicts two white dwarfs
RXJ0806.3+1527 or J0806, swirling close together...

Credit: GSFC/D. Berry

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Primordial Gravity Wave Background

- ◆ Waves stemming from the inflationary expansion of space itself
- ◆ Waves from the collision of bubble-like clumps of new matter at reheating after inflation
- ◆ Waves from the turbulent fluid mixing of the early pools of matter and radiation

2nd order Tensors from 1st order Scalars

Metric
Decomposition

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + \underbrace{\delta g_{\mu\nu}}_{\text{purely scalar d.o.f}} + \underbrace{\delta^2 g_{\mu\nu}}_{\text{purely tensor d.o.f}}$$

S. Matarrese, O. Pantano, and D. Saez, PRD, 47, 1311 (1993)

K. N. Ananda, C. Clarkson, and D. Wands, PRD, 75, 123518 (2007)

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$$\Phi'' + 3aH(1 + c_s^2)\Phi' + c_s^2 k^2 \Phi = 0$$

SCALAR

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SCALAR

TENSOR

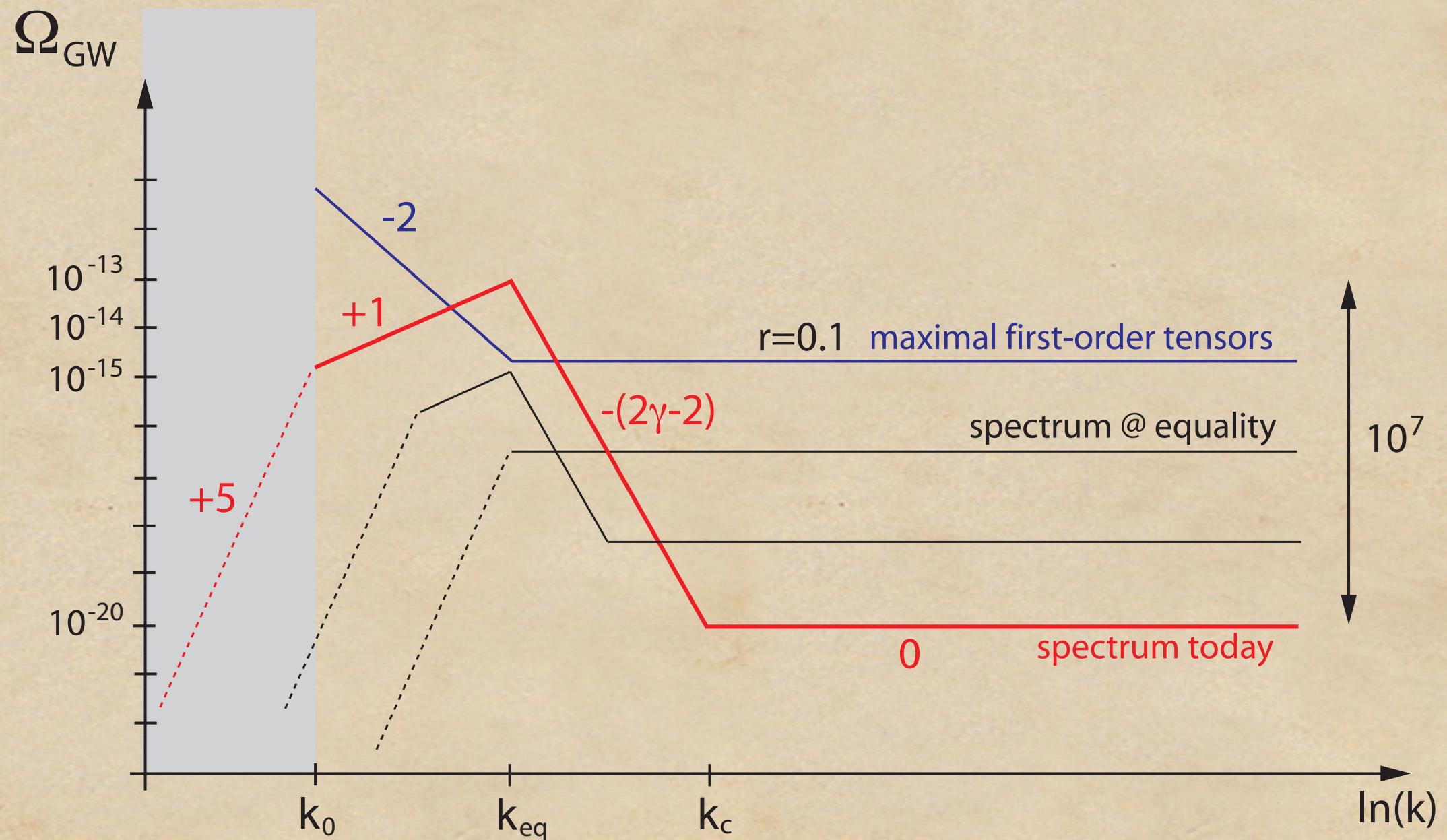
$$h''_{ij} + 2aHh'_{ij} - \nabla^2 h_{ij} = -4\hat{T}_{ij}^{lm} S_{lm}$$

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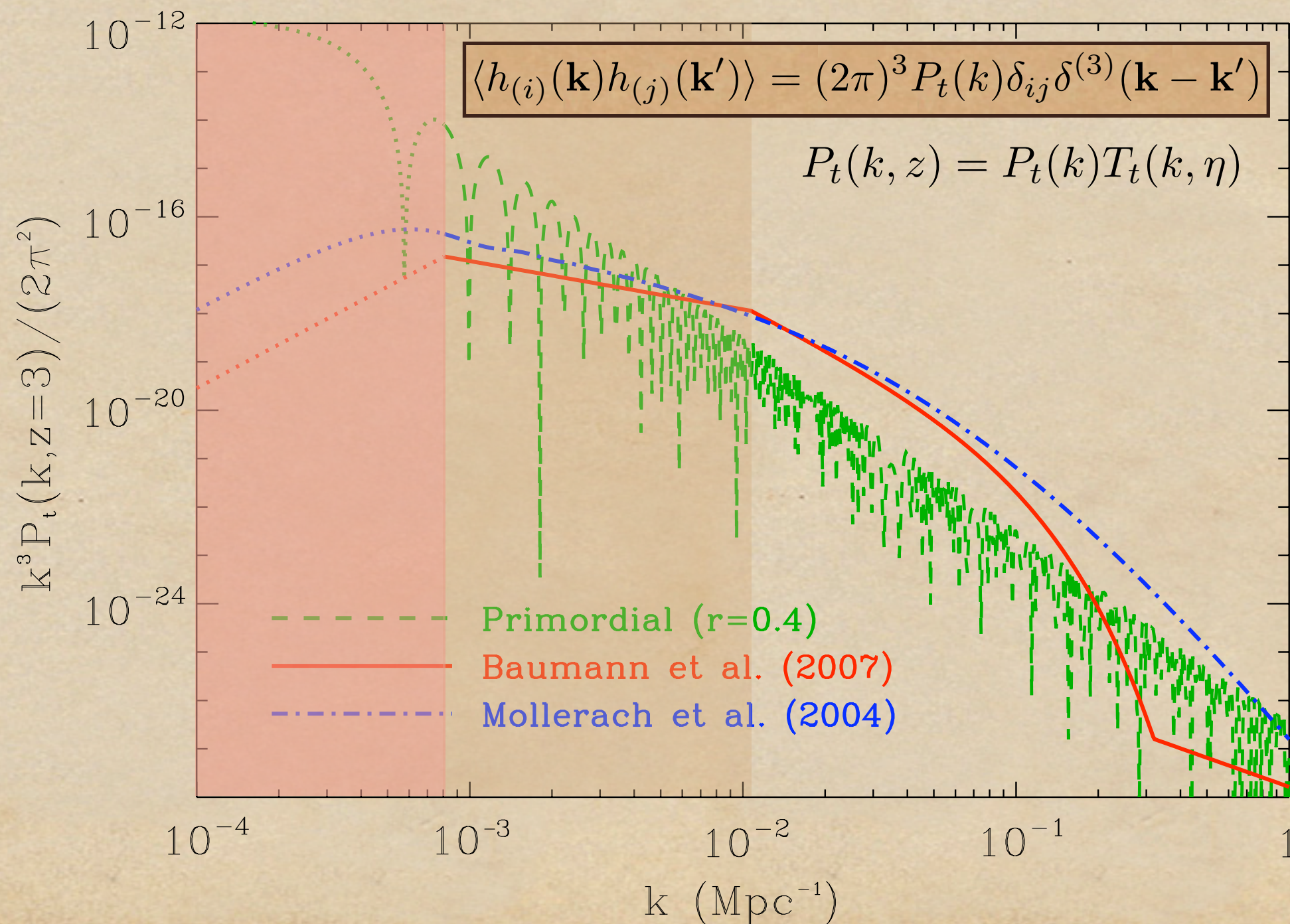
D. Baumann, P. Steinhardt, K. Takahashi, and K. Ichiki, PRD, 76, 084019 (2007)

Relative Strengths



D. Baumann, P. Steinhardt, K. Takahashi, and K. Ichiki, PRD, 76, 084019 (2007)

How Do the Power Spectra Look Like?

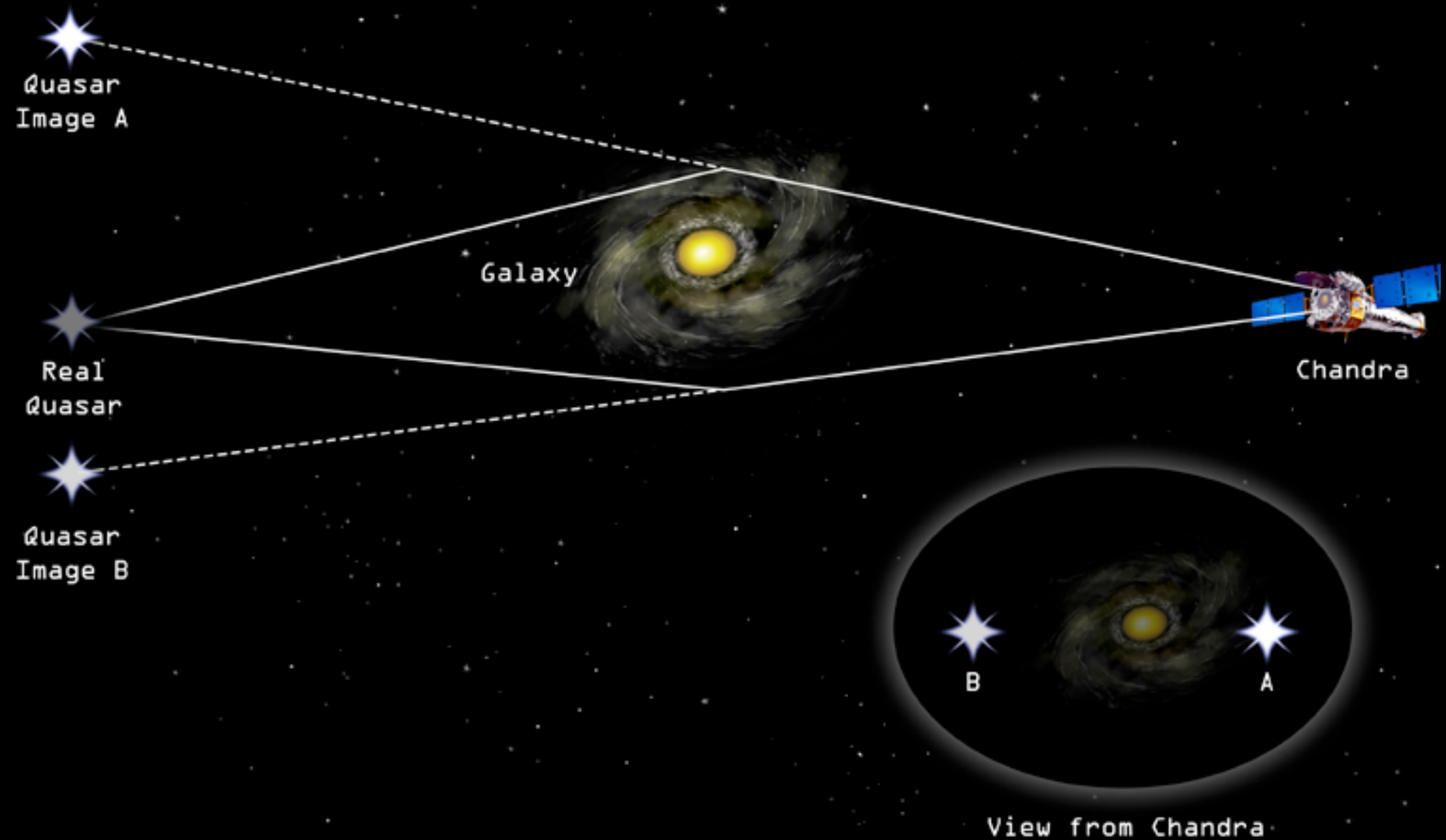


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Gravitational Lensing

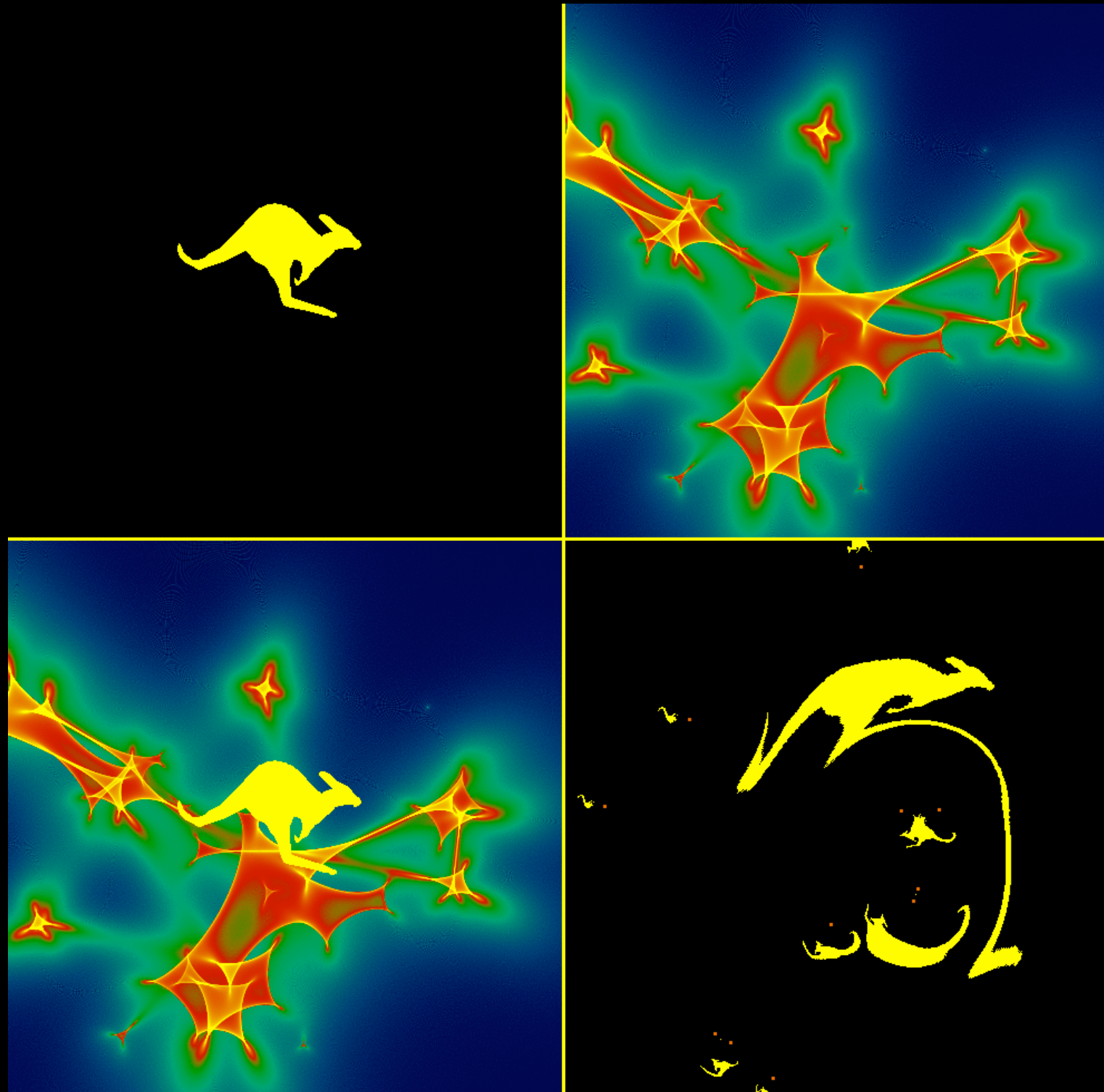


Gravitational Lensing

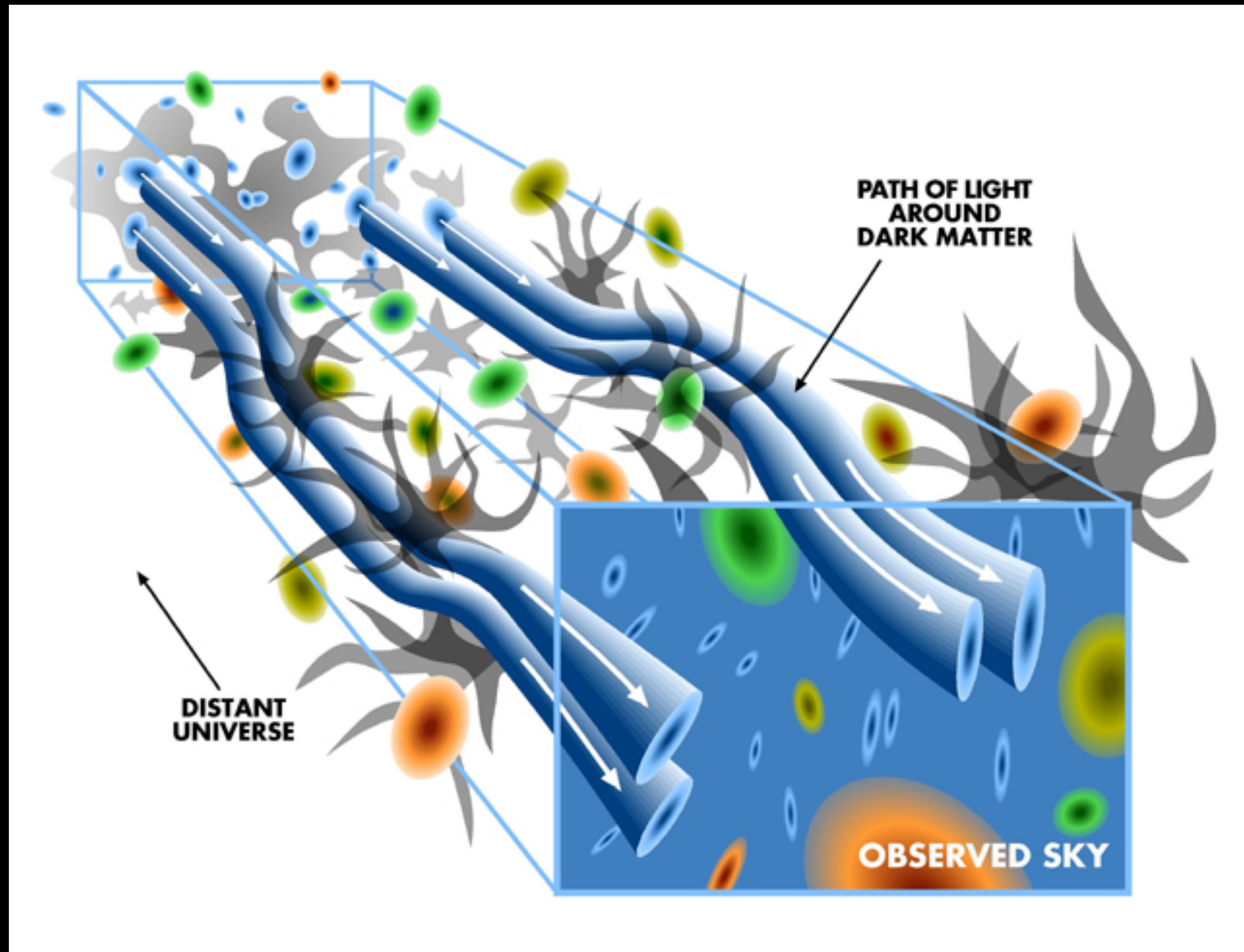
Lensing Galaxy



Gravitational Lensing



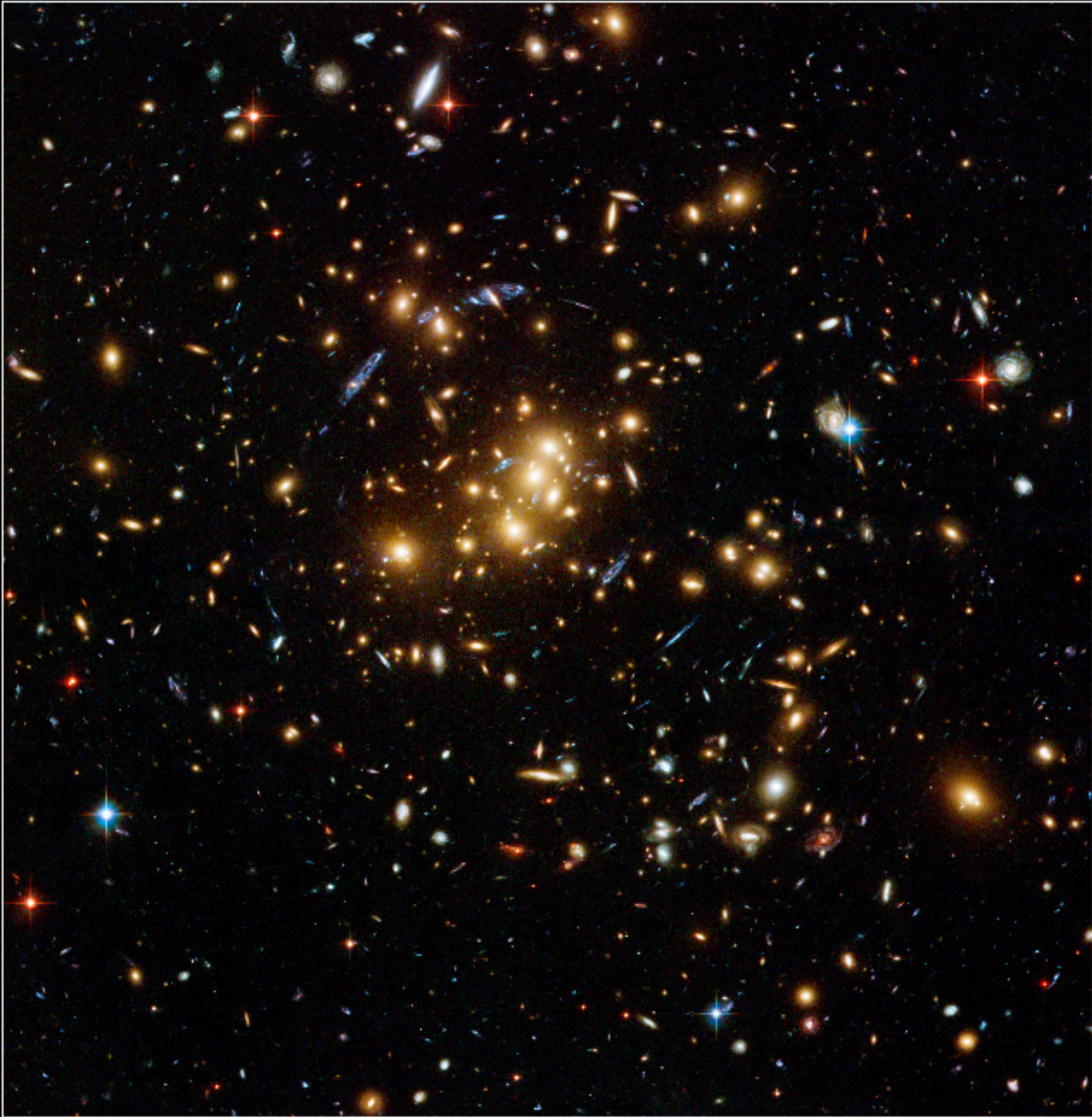
Gravitational Lensing



Reconstruction of Dark Matter Distribution from Observation

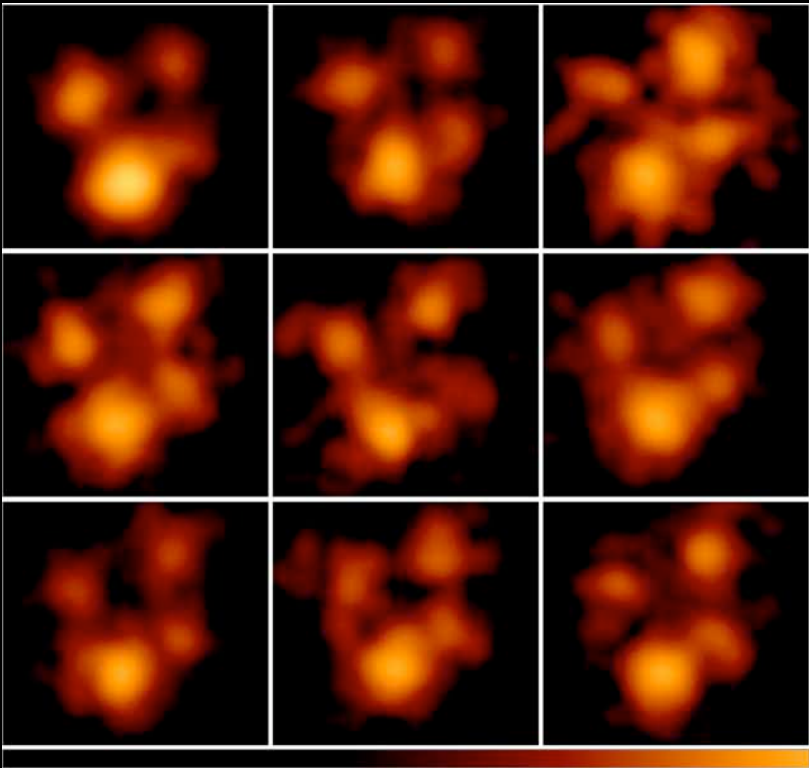
Galaxy Cluster CI 0024+17 (ZwCl 0024+1652)

HST • ACS/WFC

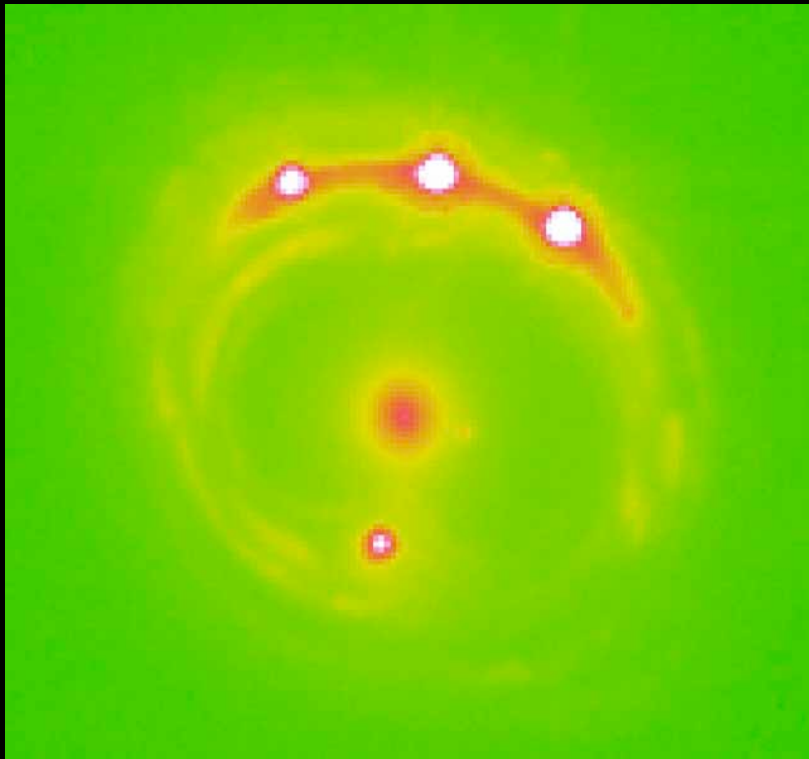


NASA, ESA, and M.J. Jee (Johns Hopkins University)

STScI-PRC07-17b



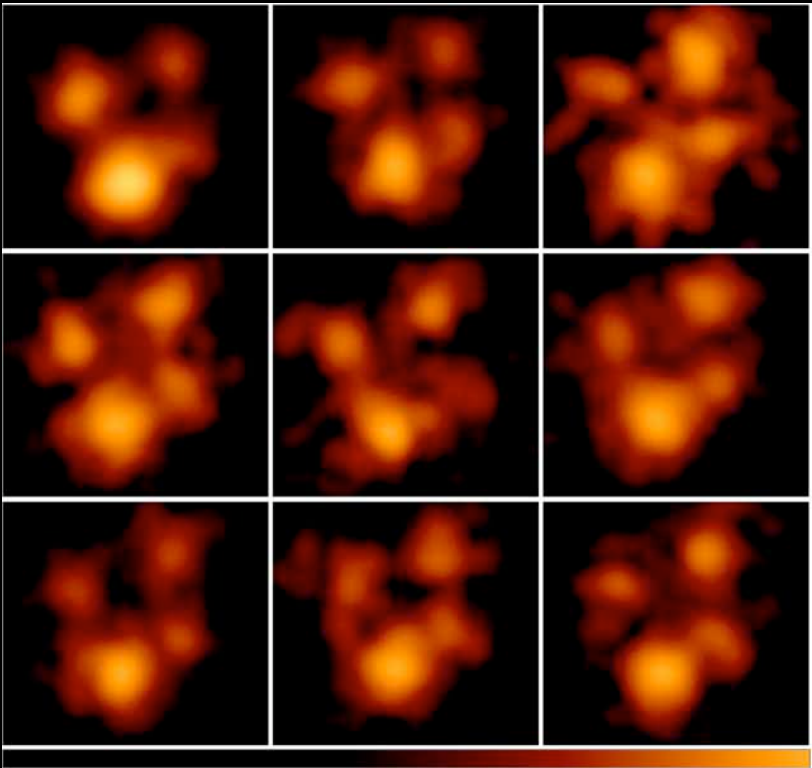
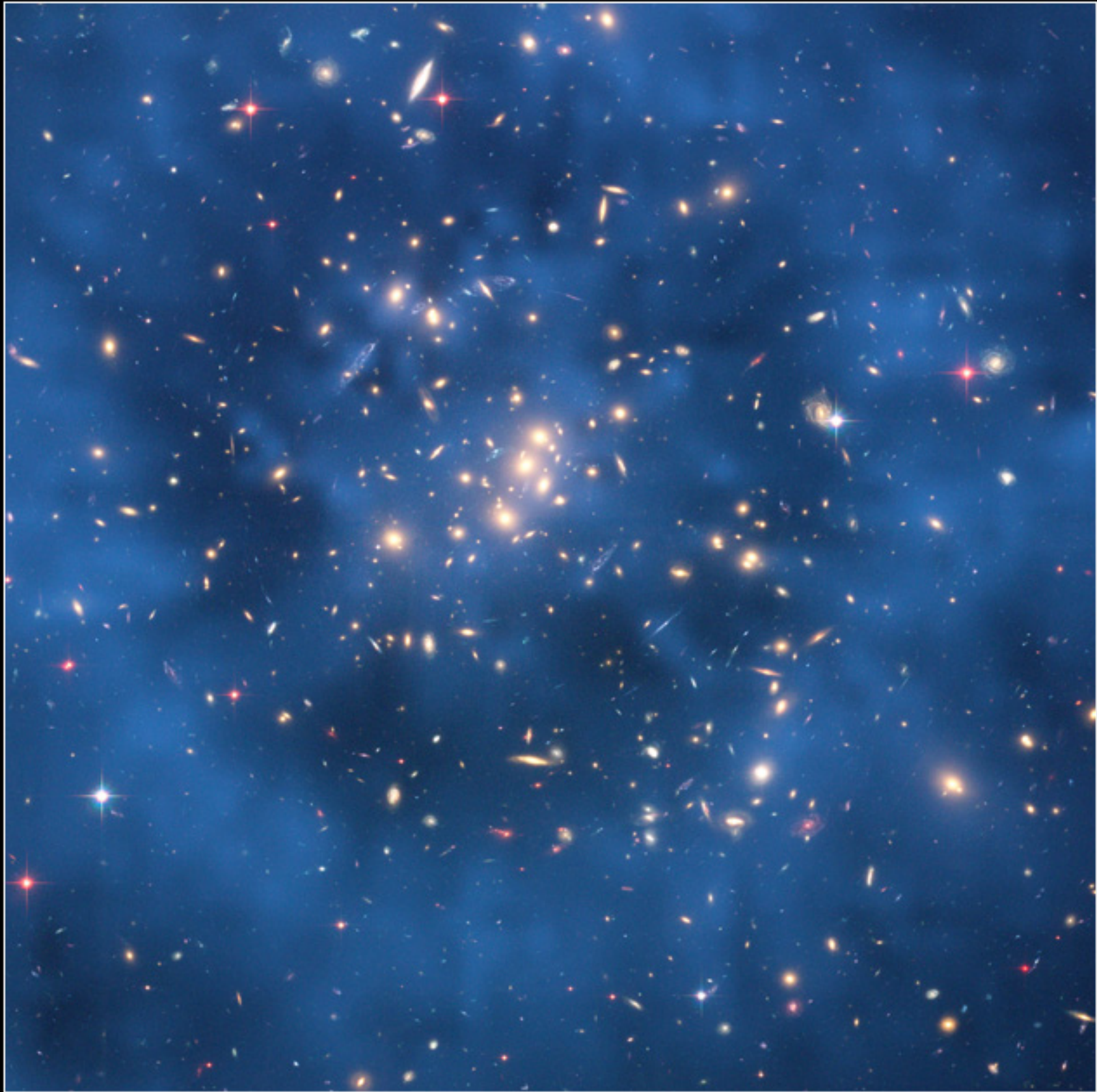
X-Ray images of Quasar Q2237+0305
(Mosaic courtesy: Ohio State University)



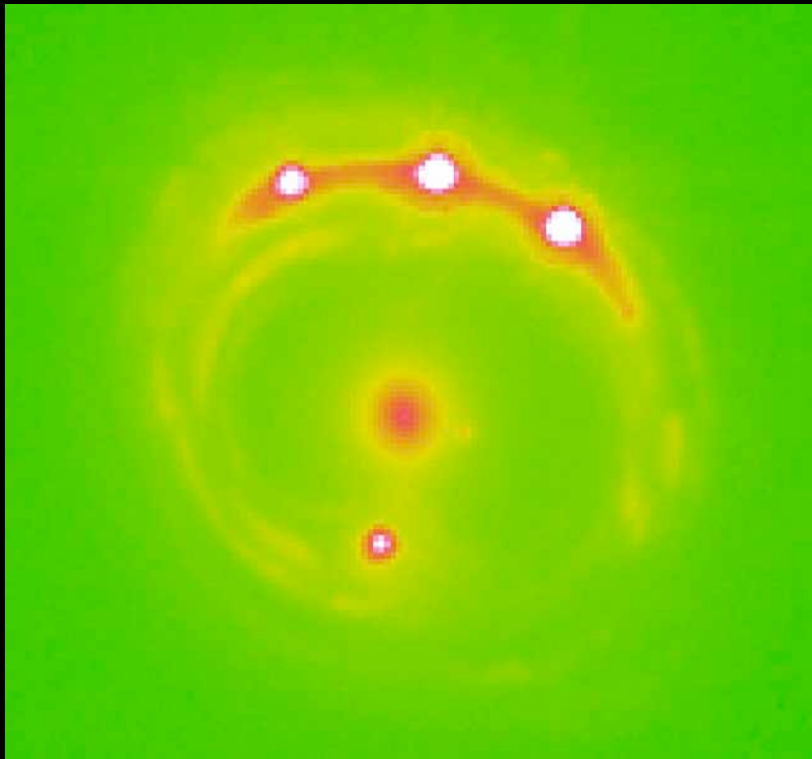
Optical images of Quasar RXJ1131-1231
(courtesy: Ohio State University)

Reconstruction of Dark Matter Distribution from Observation

Dark Matter Ring in Cl 0024+17 (ZwCl 0024+1652) *HST* • ACS/WFC



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Dark Matter Expose: Bullet Cluster

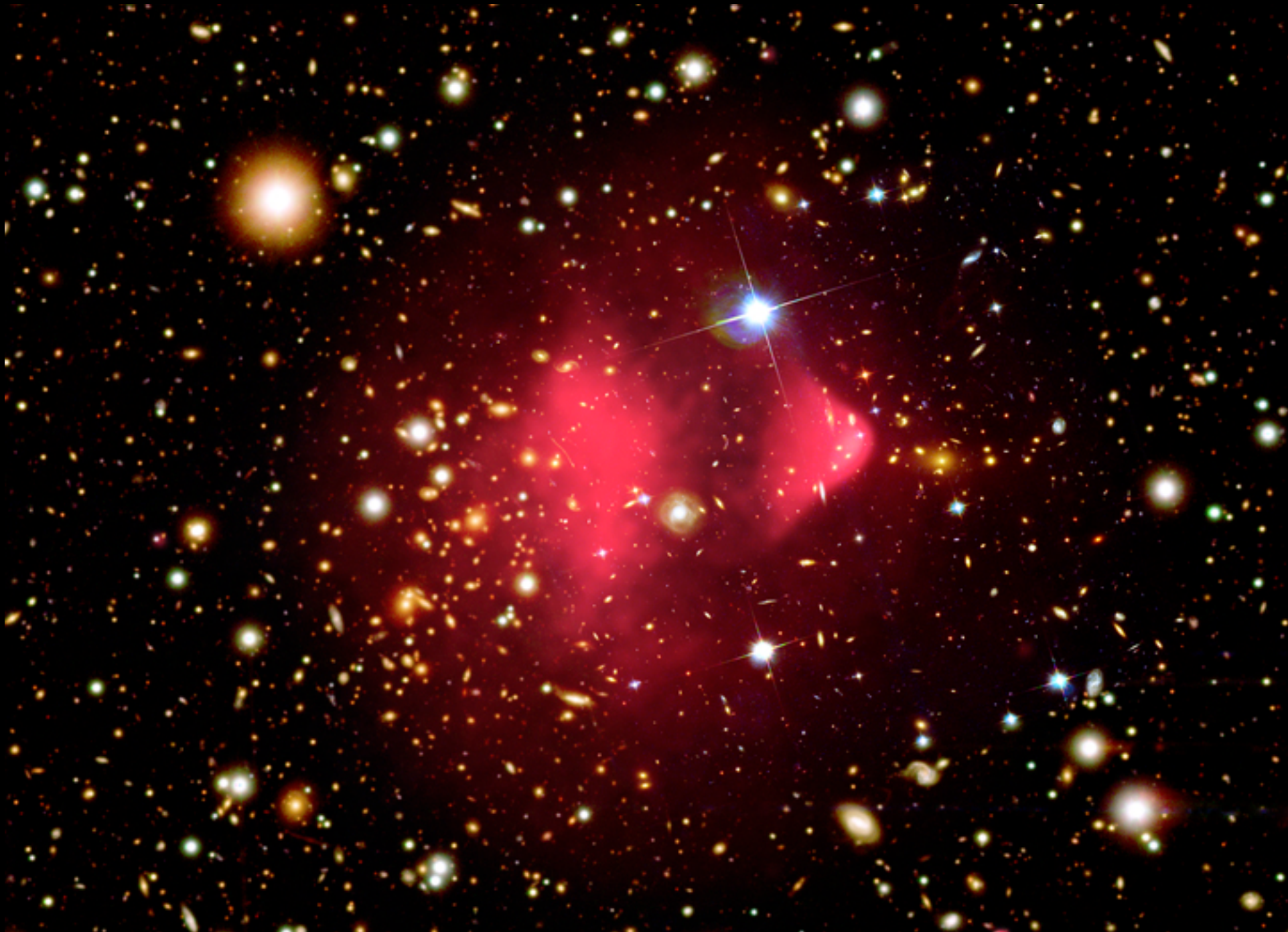
OPTICAL



X-ray: NASA/CXC/CfA/M.Markevitch et al.; Optical: NASA/STScI; Magellan/U.Arizona/
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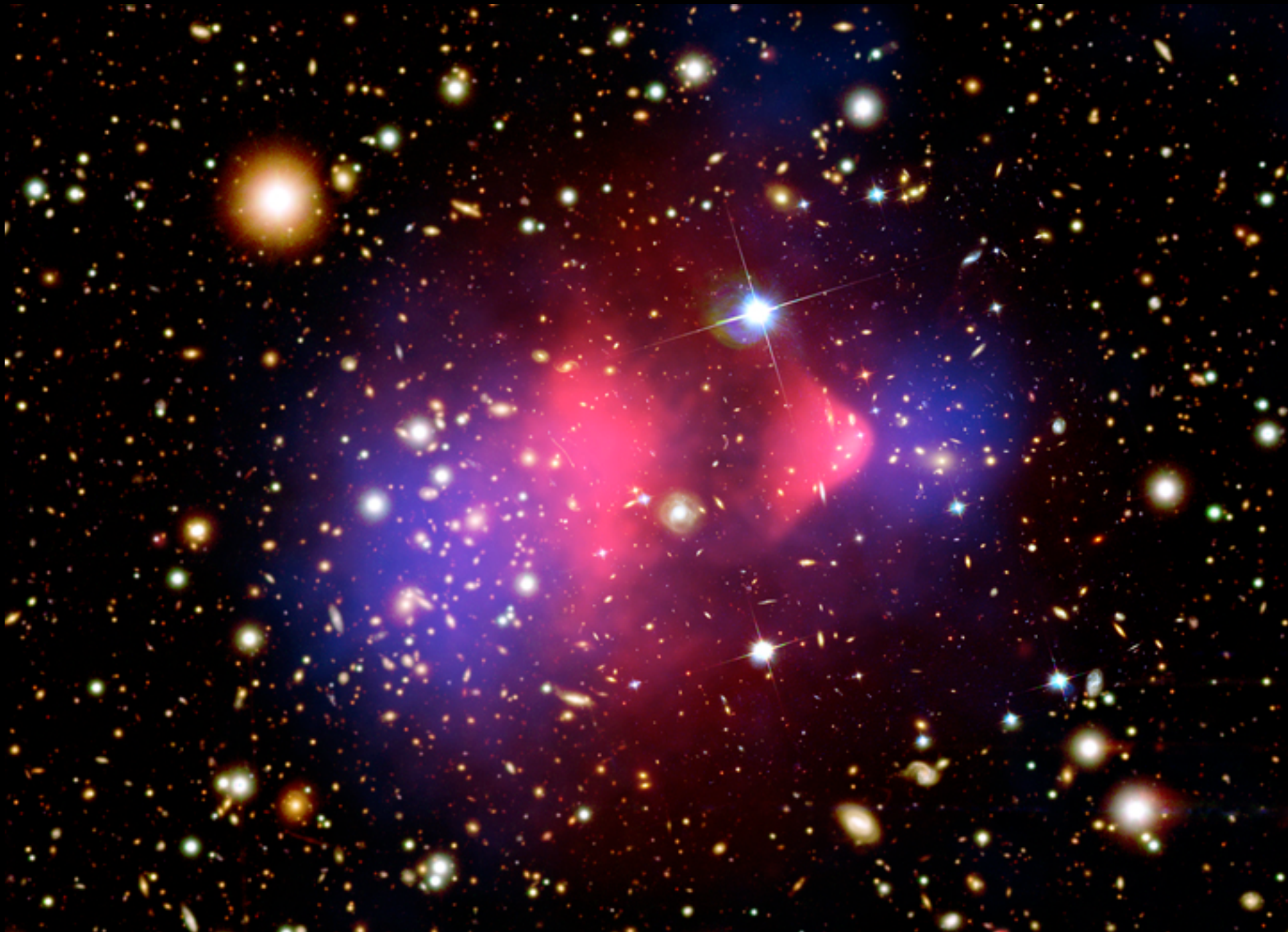


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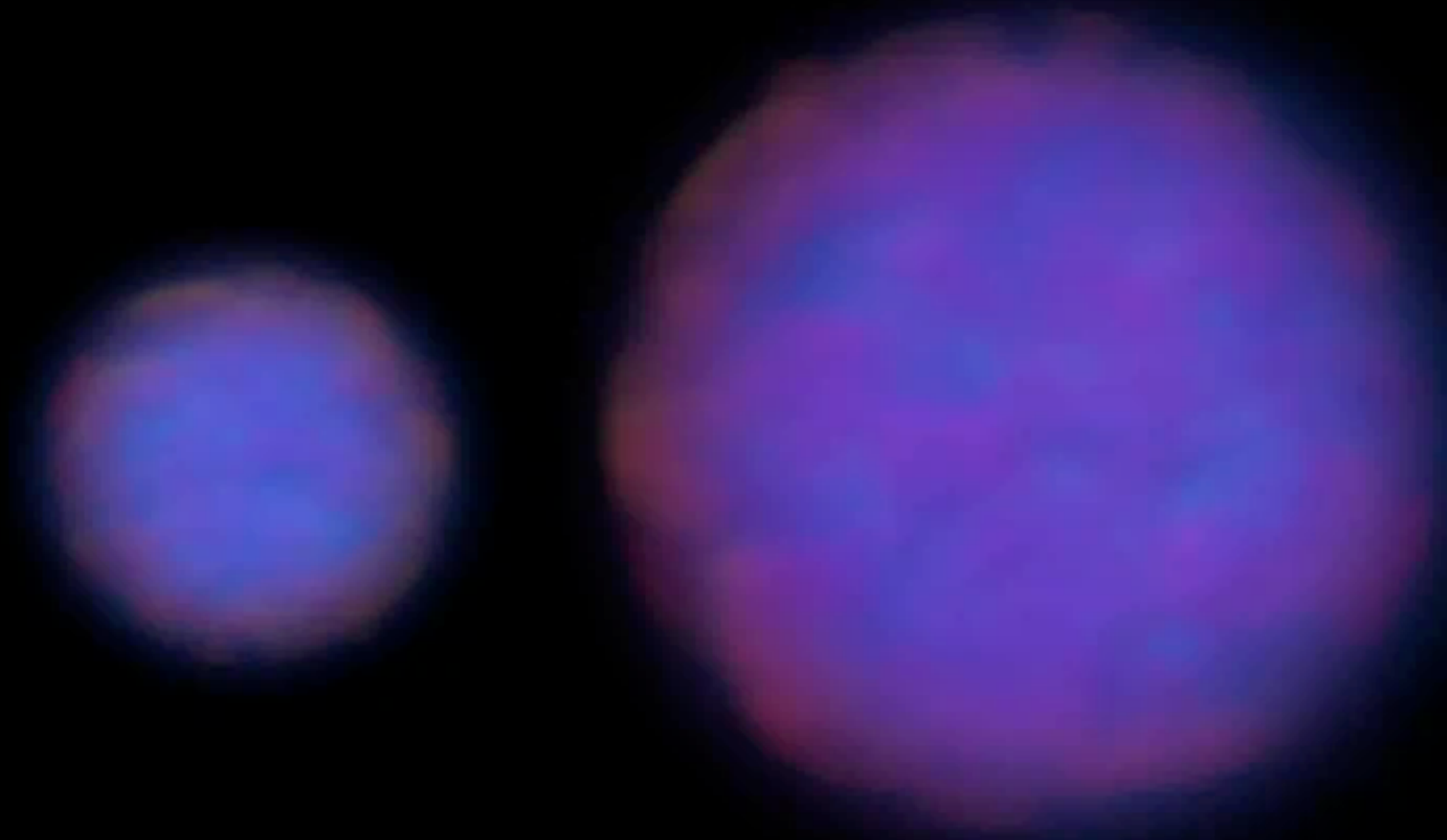
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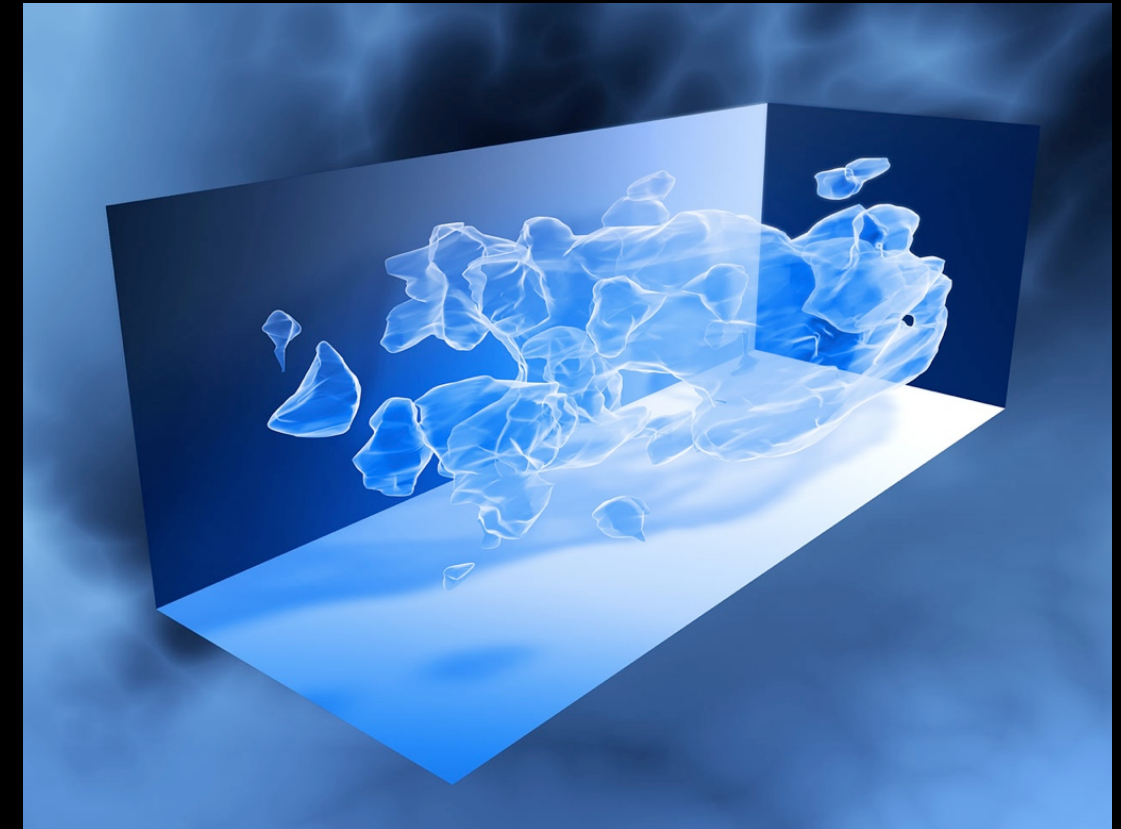
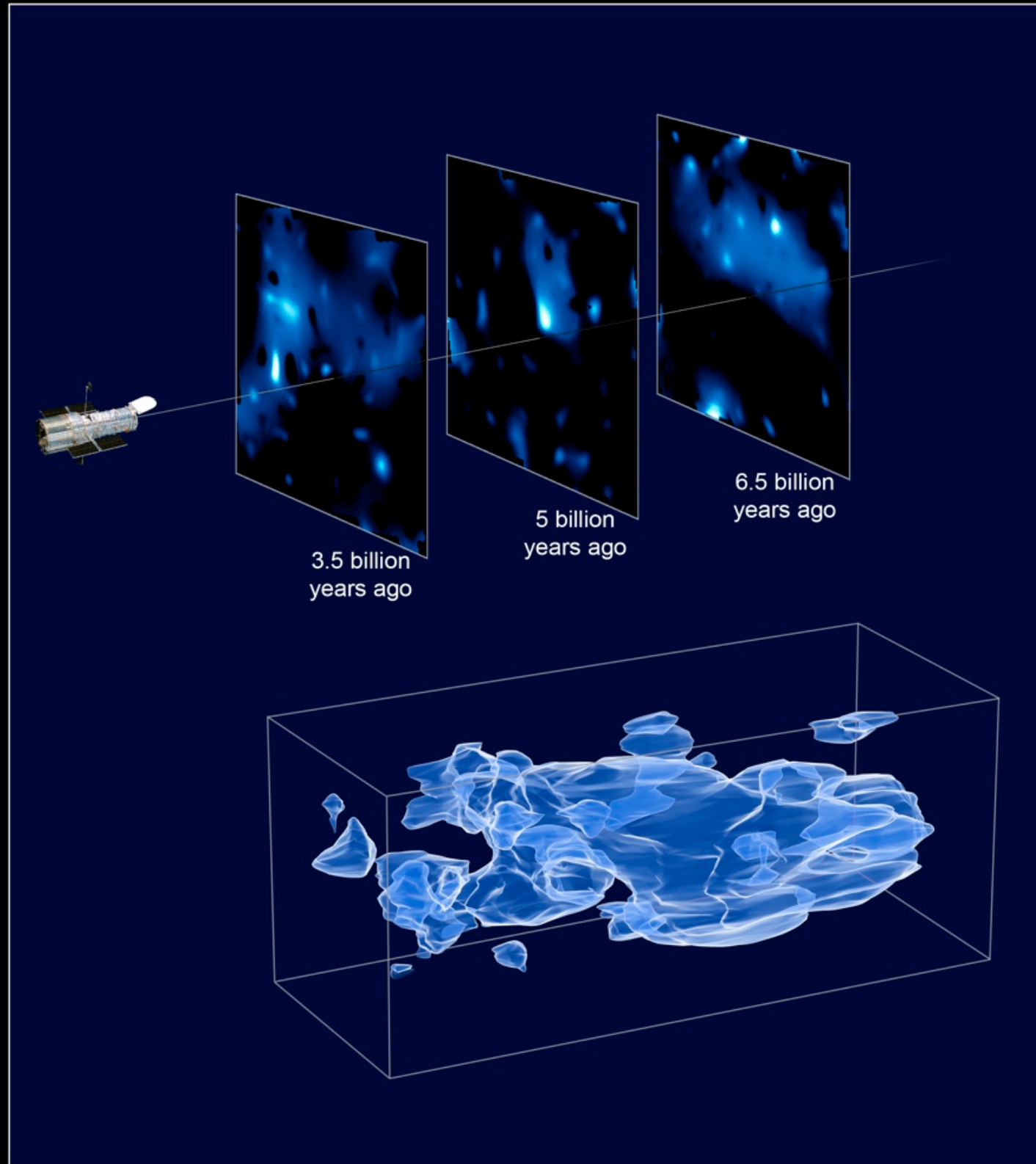
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Distribution of Dark Matter

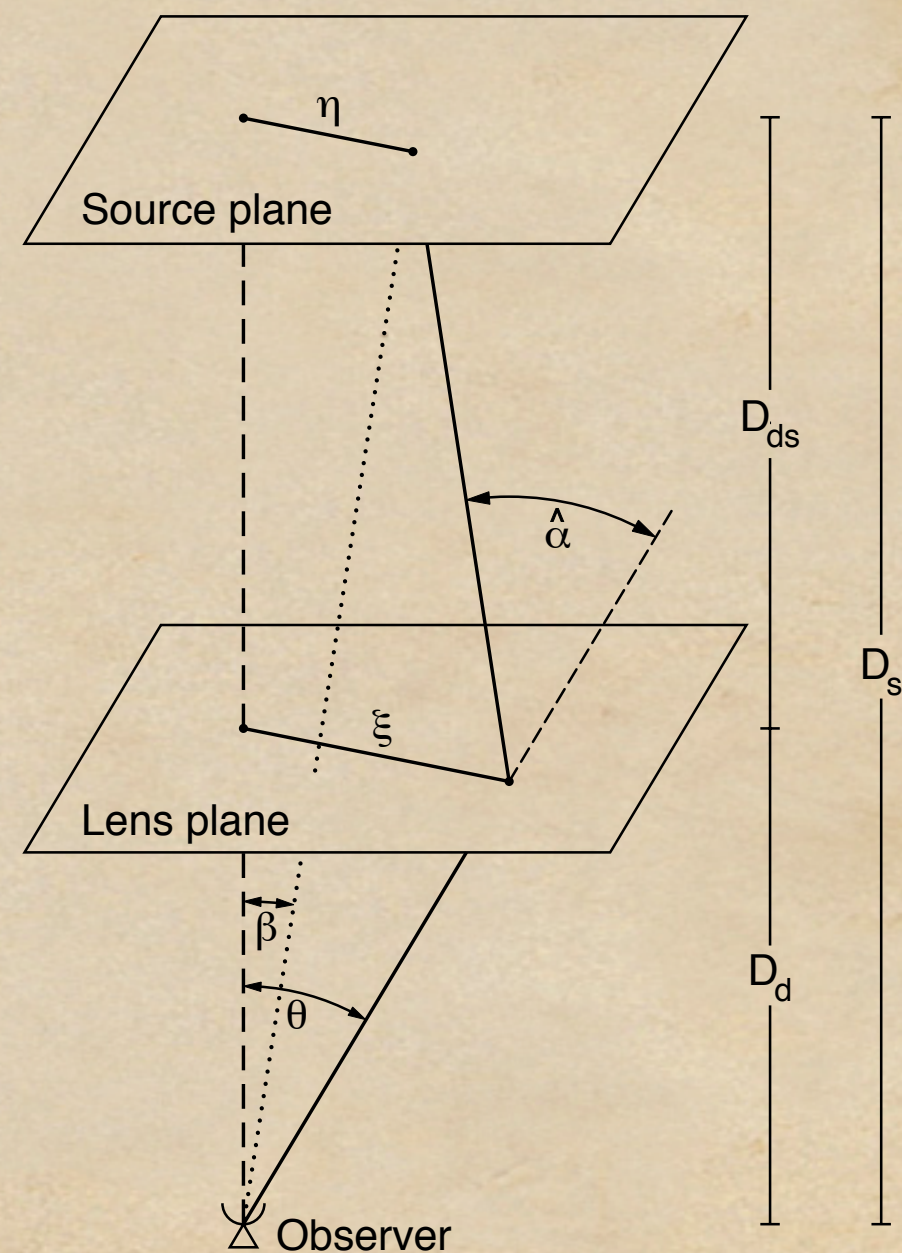
HST ■ ACS/WFC



Gravitational Lensing 201

The Lens Equation:

$$\boxed{\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})} \quad \alpha(\vec{\theta}) = \frac{D_{ds}}{D_s} \hat{\alpha}(D_d \vec{\theta})$$



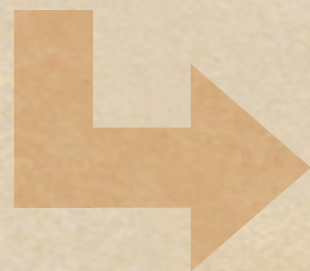
Credit: Bartelmann and Schneider 2001

Gravitational Lensing 201

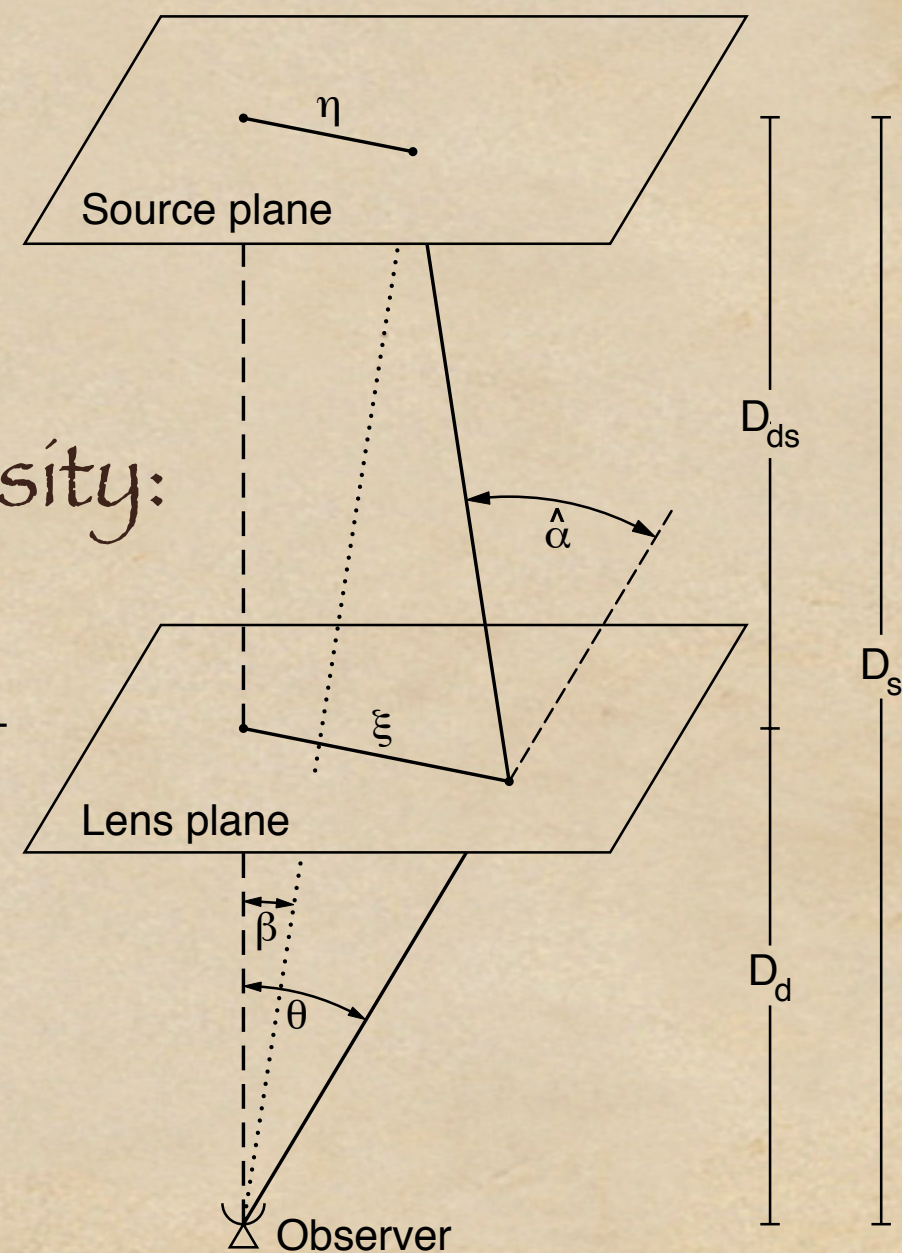
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Dimensionless Surface Mass Density:



$$\kappa(\vec{\theta}) = \frac{\Sigma(D_d \vec{\theta})}{\Sigma_{cr}}$$



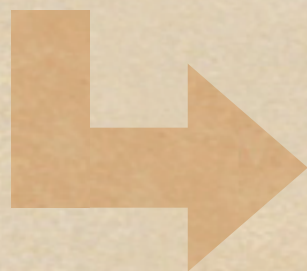
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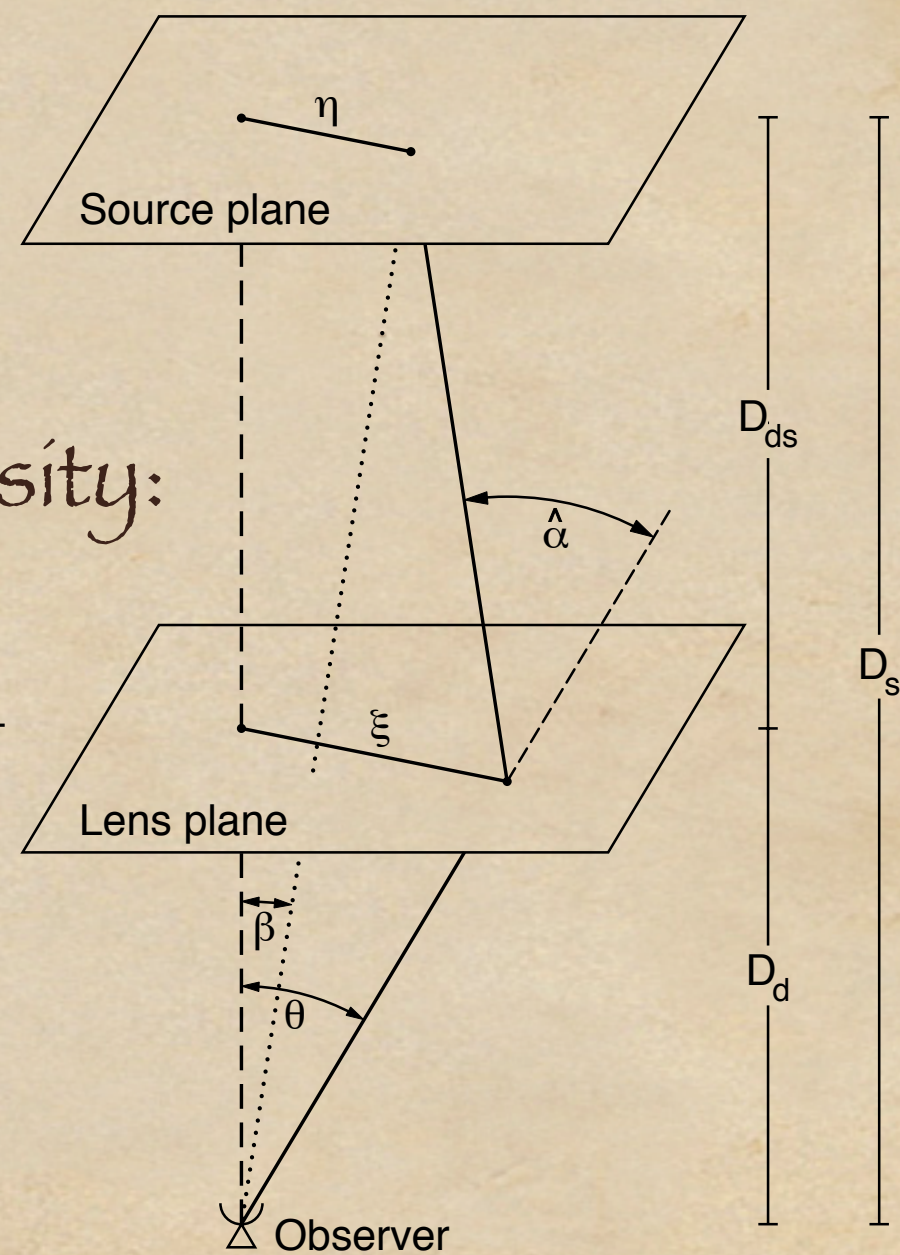
Dimensionless Surface Mass Density:



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Deflection angle:

$$\vec{\alpha}(\vec{\theta}) = \frac{1}{\pi} \int_{R^2} d^2 \theta' \kappa(\vec{\theta}') \frac{\vec{\theta} - \vec{\theta}'}{|\vec{\theta} - \vec{\theta}'|^2}$$



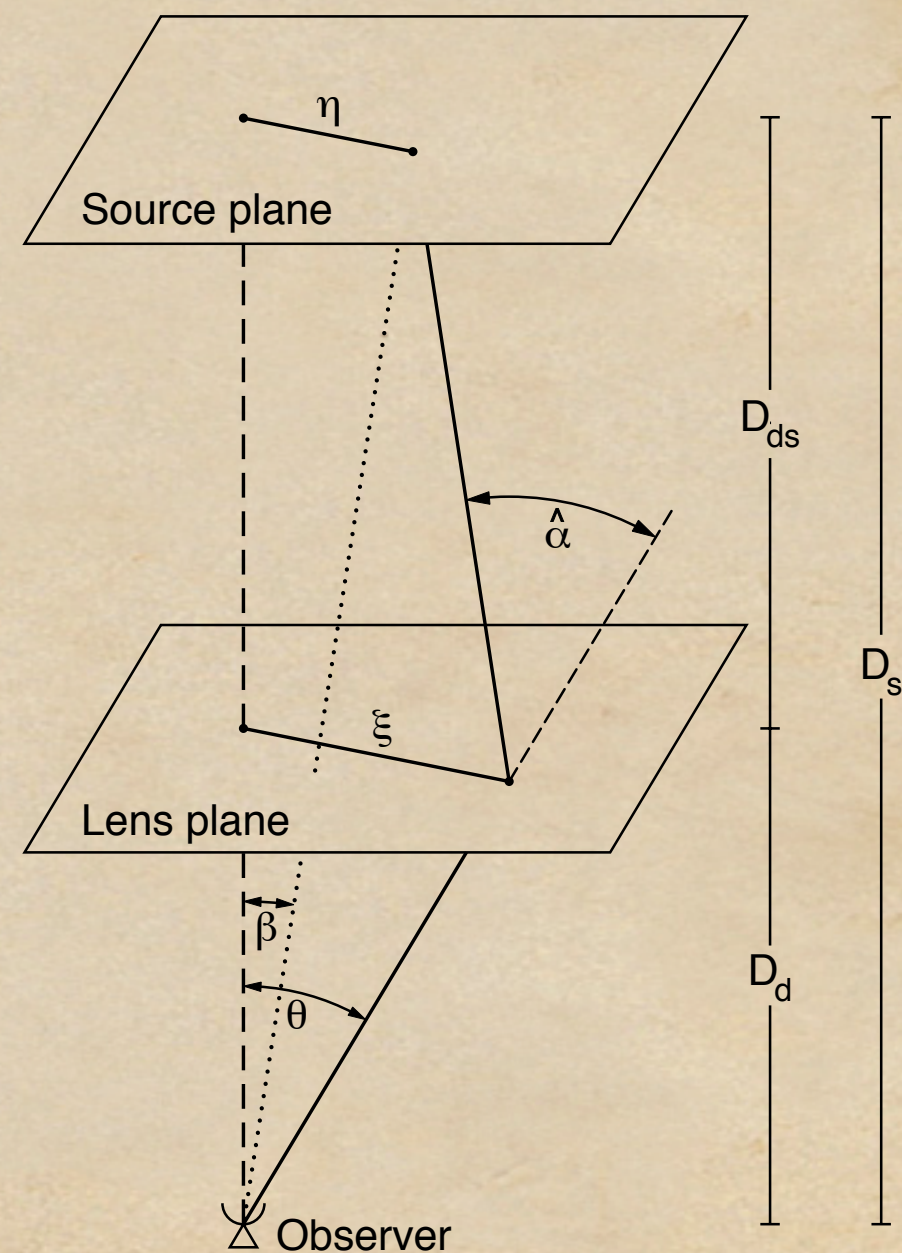
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Gravitational Lensing 201

The Lens Equation:

$$\boxed{\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})} \quad \alpha(\vec{\theta}) = \frac{D_{ds}}{D_s} \hat{\alpha}(D_d \vec{\theta})$$

$$\vec{\alpha}(\vec{\theta}) = \frac{1}{\pi} \int_{R^2} d^2\theta' \kappa(\vec{\theta}') \frac{\vec{\theta} - \vec{\theta}'}{|\vec{\theta} - \vec{\theta}'|^2}$$



Credit: Bartelmann and Schneider 2001

Gravitational Lensing 201

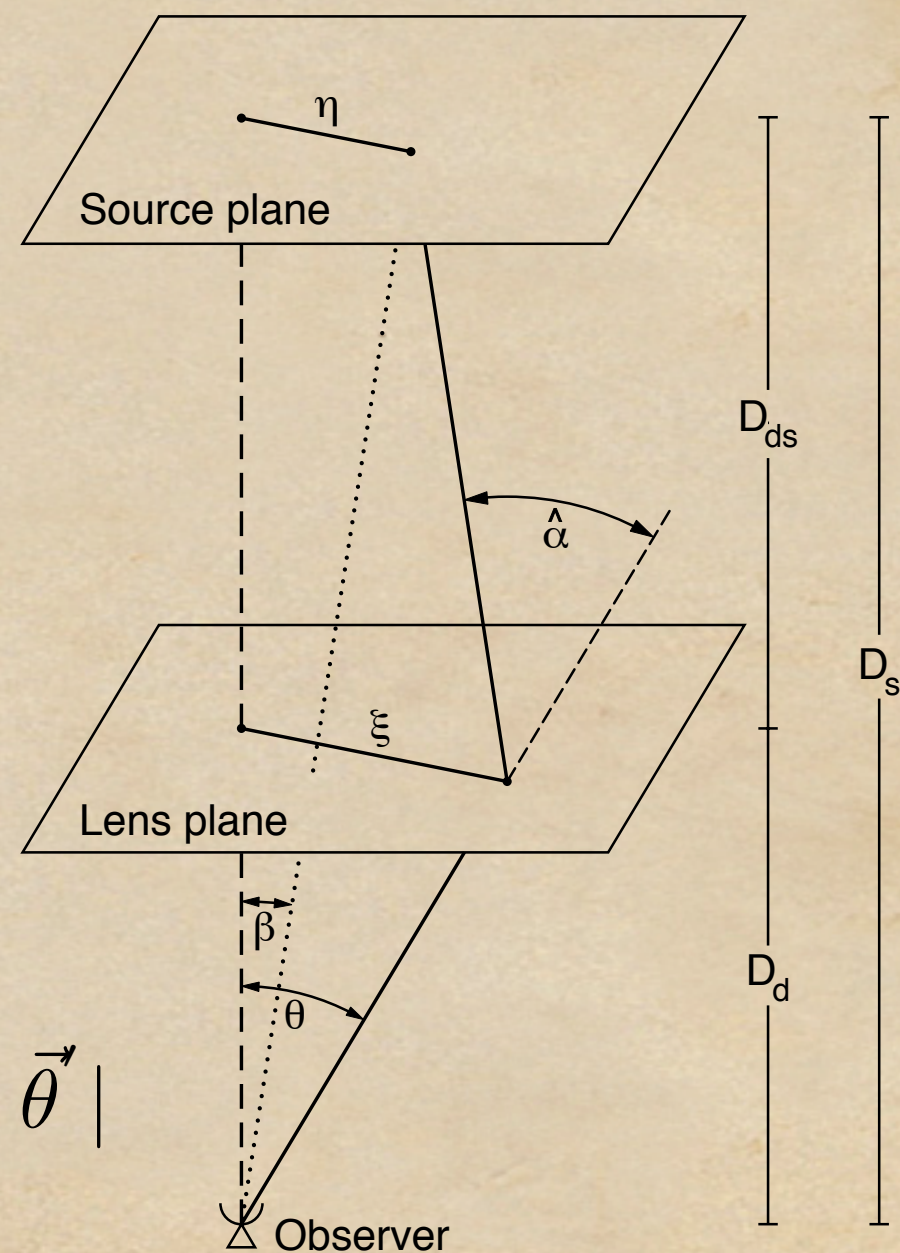
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$$\alpha(\vec{\theta}) = \nabla \phi$$

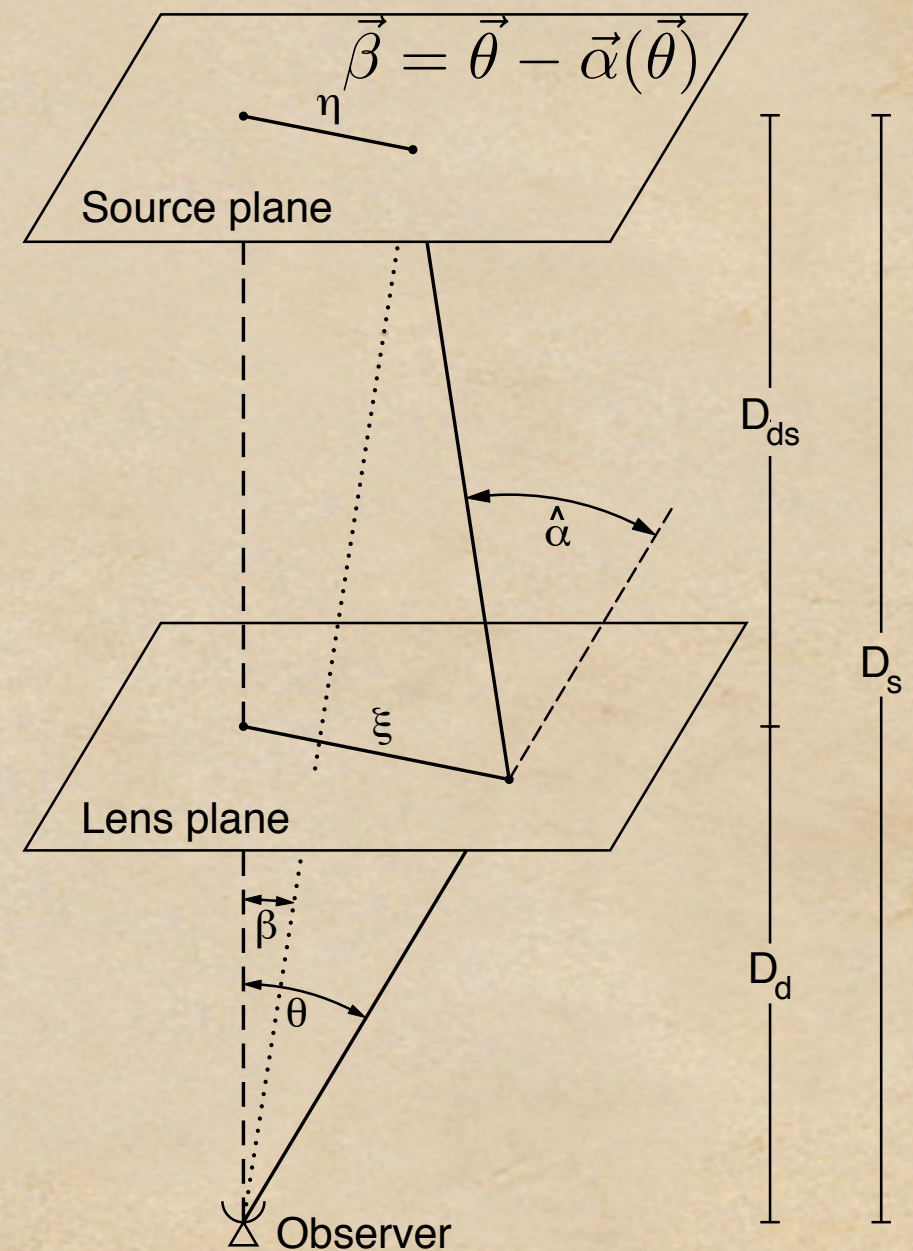
$$\phi(\vec{\theta}) = \frac{1}{\pi} \int_{R^2} d^2\theta' \kappa(\vec{\theta}') \ln |\vec{\theta} - \vec{\theta}'|$$



Credit: Bartelmann and Schneider 2001

Gravitational Lensing 201

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$



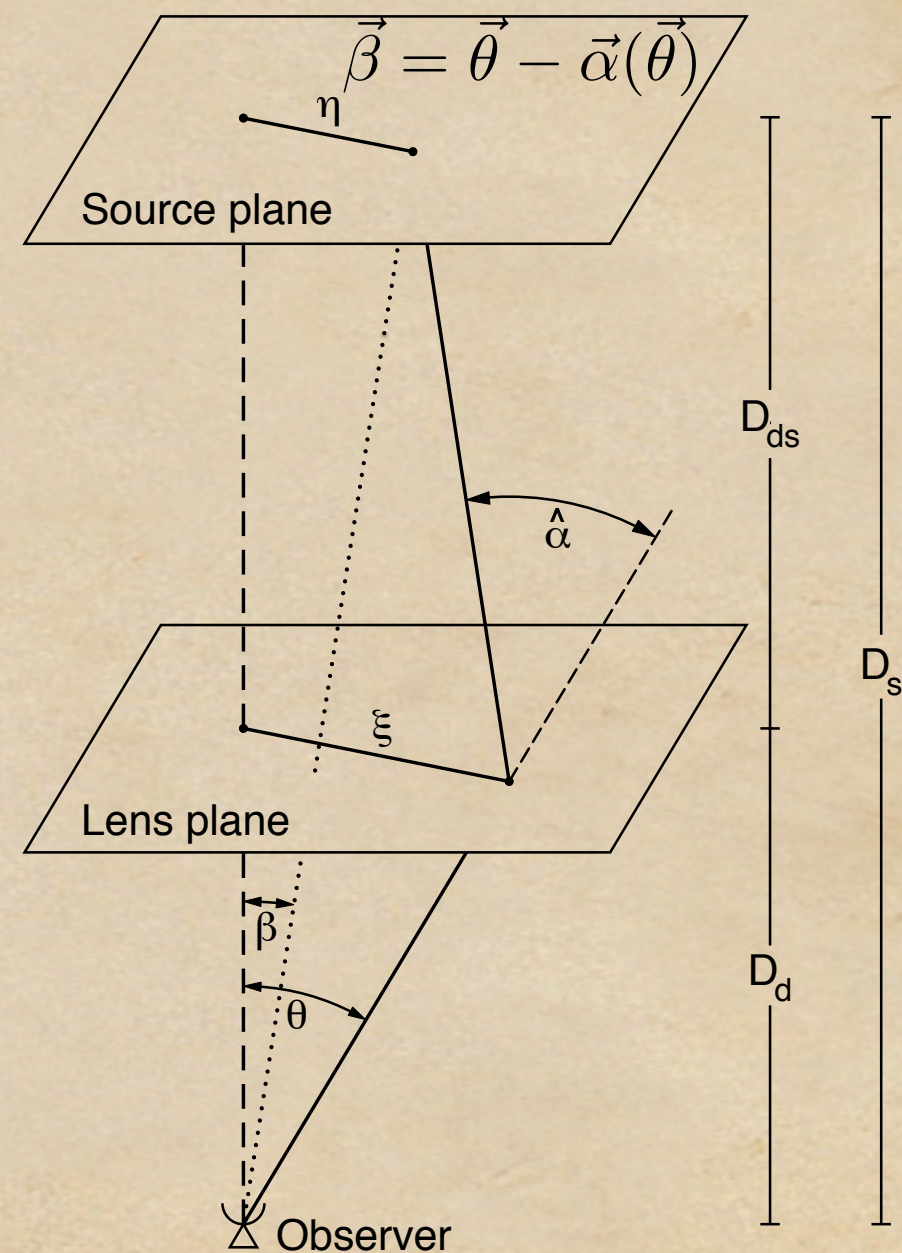
Credit: Bartelmann and Schneider 2001

Gravitational Lensing 201

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

Distortion of images by Jacobian:

$$\mathcal{A}(\vec{\theta}) = \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \left(\delta_{ij} - \frac{\partial^2 \phi(\vec{\theta})}{\partial \theta_i \partial \theta_j} \right)$$



Credit: Bartelmann and Schneider 2001

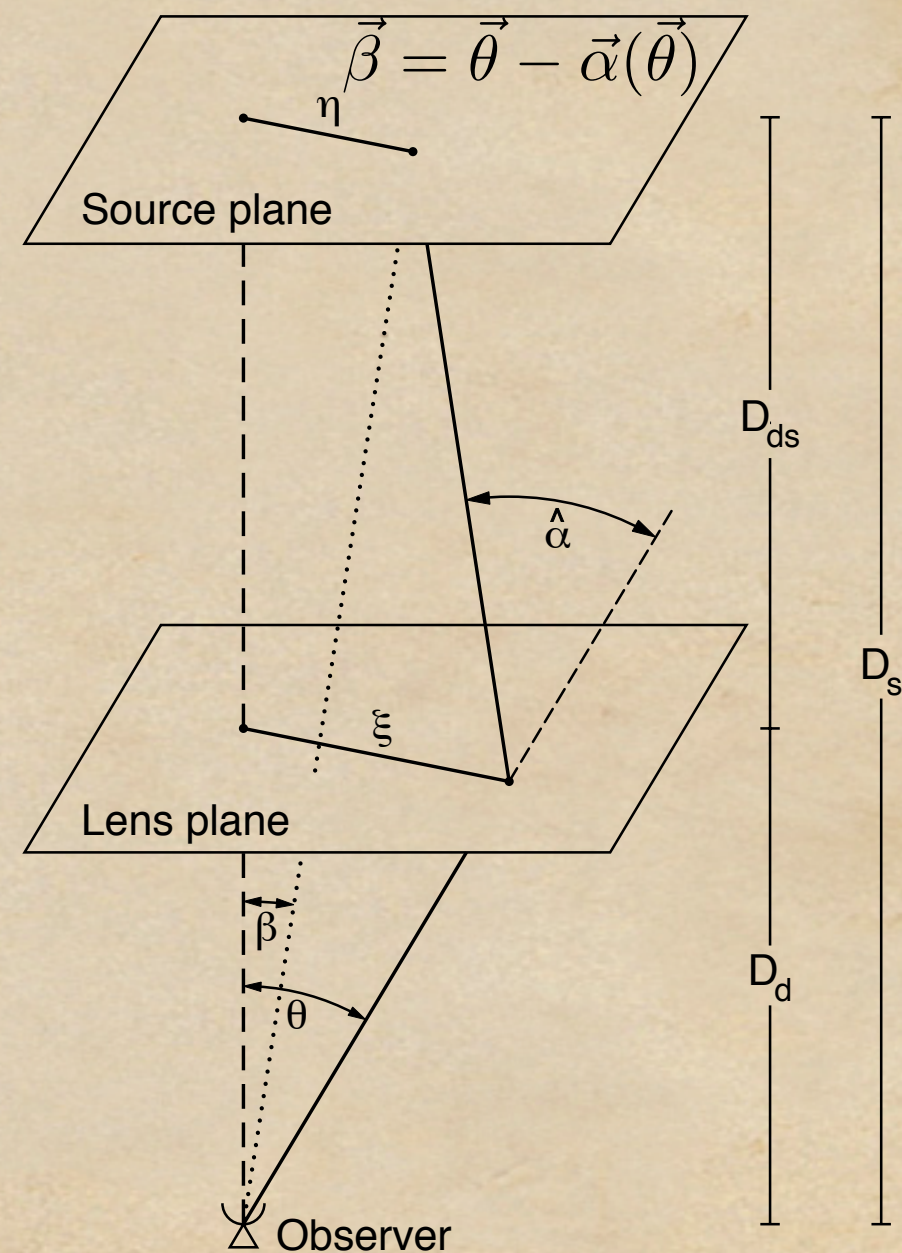
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$$\mathcal{A}(\vec{\theta}) = (1 - \kappa) \delta_{ij} - \gamma_{ij}$$



Credit: Bartelmann and Schneider 2001

Gravitational Lensing 201

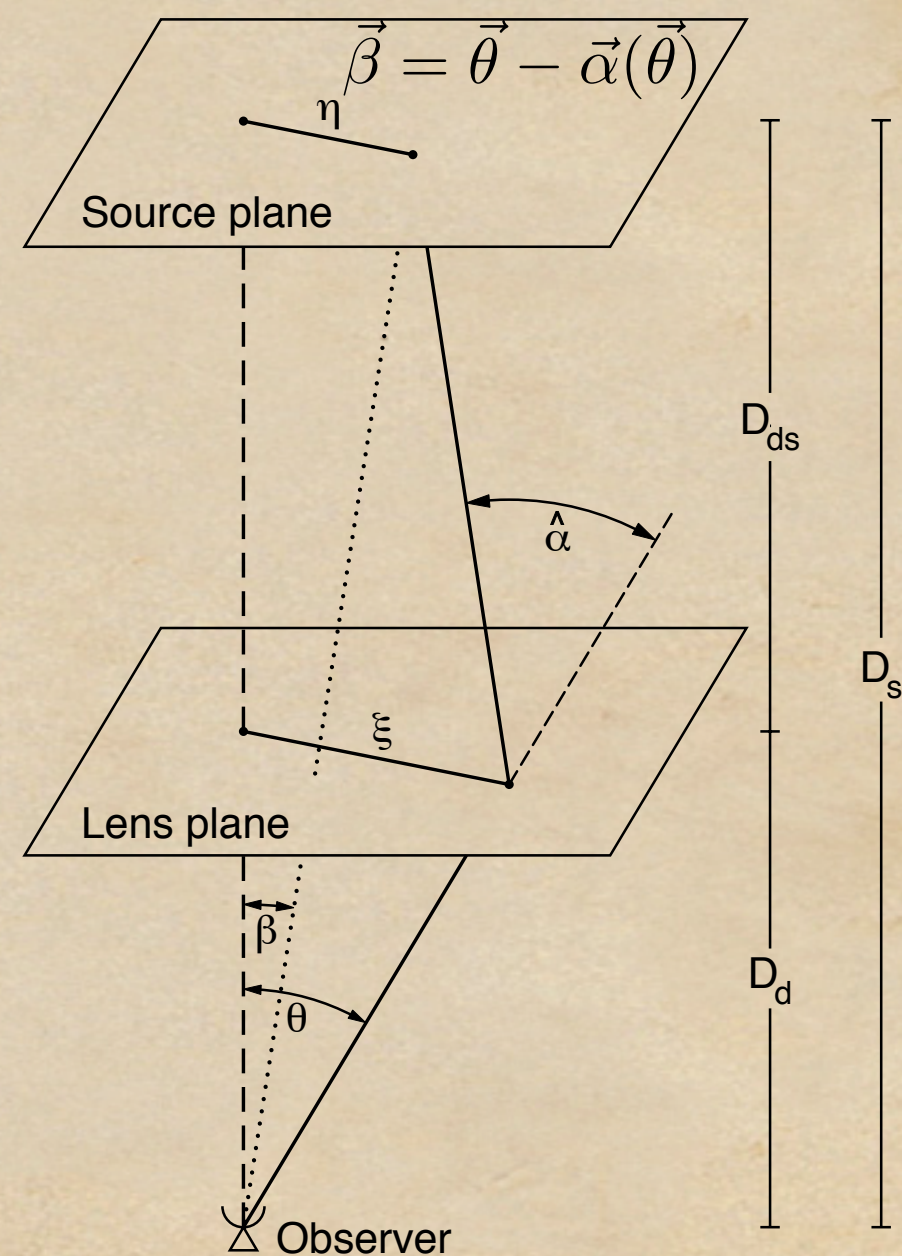
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$$\mathcal{A}(\vec{\theta}) = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}$$



Credit: Bartelmann and Schneider 2001

Gravitational Lensing 201

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

Distortion of images by Jacobian:

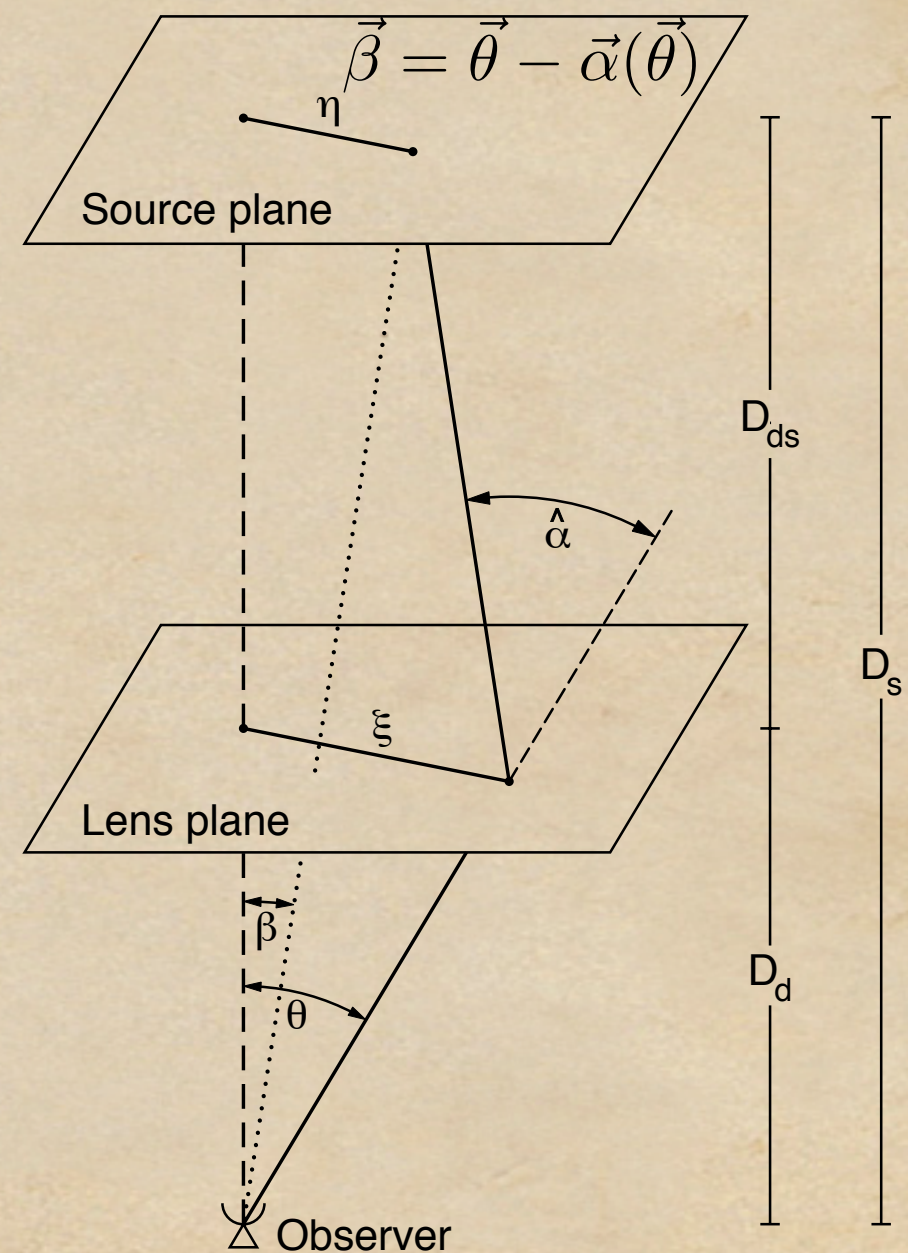
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$$\gamma \equiv \gamma_1 + i\gamma_2 = |\gamma| e^{2i\varphi}$$

$$\gamma_1 = \frac{1}{2}(\psi_{,11} - \psi_{,22}), \quad \gamma_2 = \psi_{,12}$$

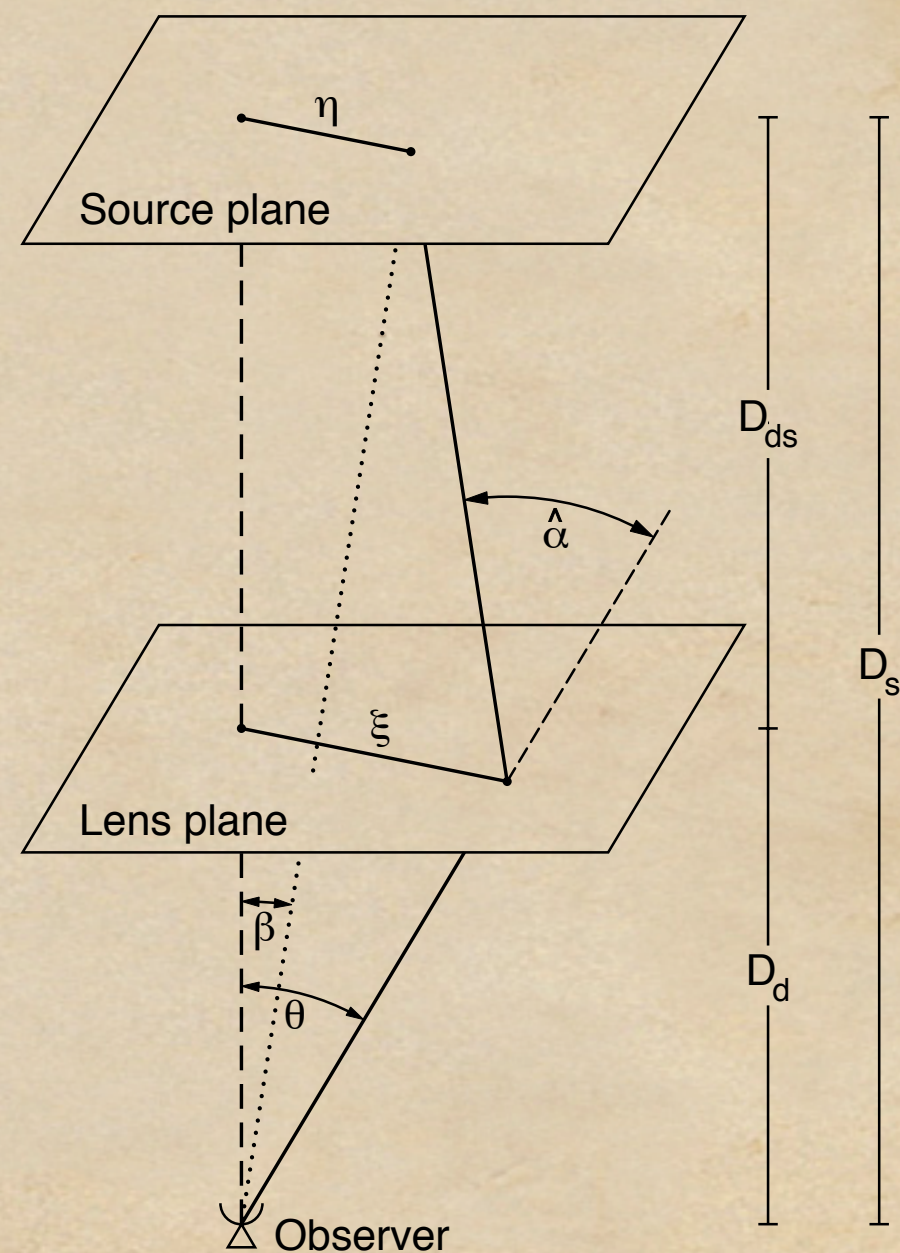


Credit: Bartelmann and Schneider 2001

Gravitational Lensing 201

But Did we Miss Anything???

$$A_{ij} = \frac{\partial x_i^S}{\partial x_j^I}$$



Credit: Bartelmann and Schneider 2001

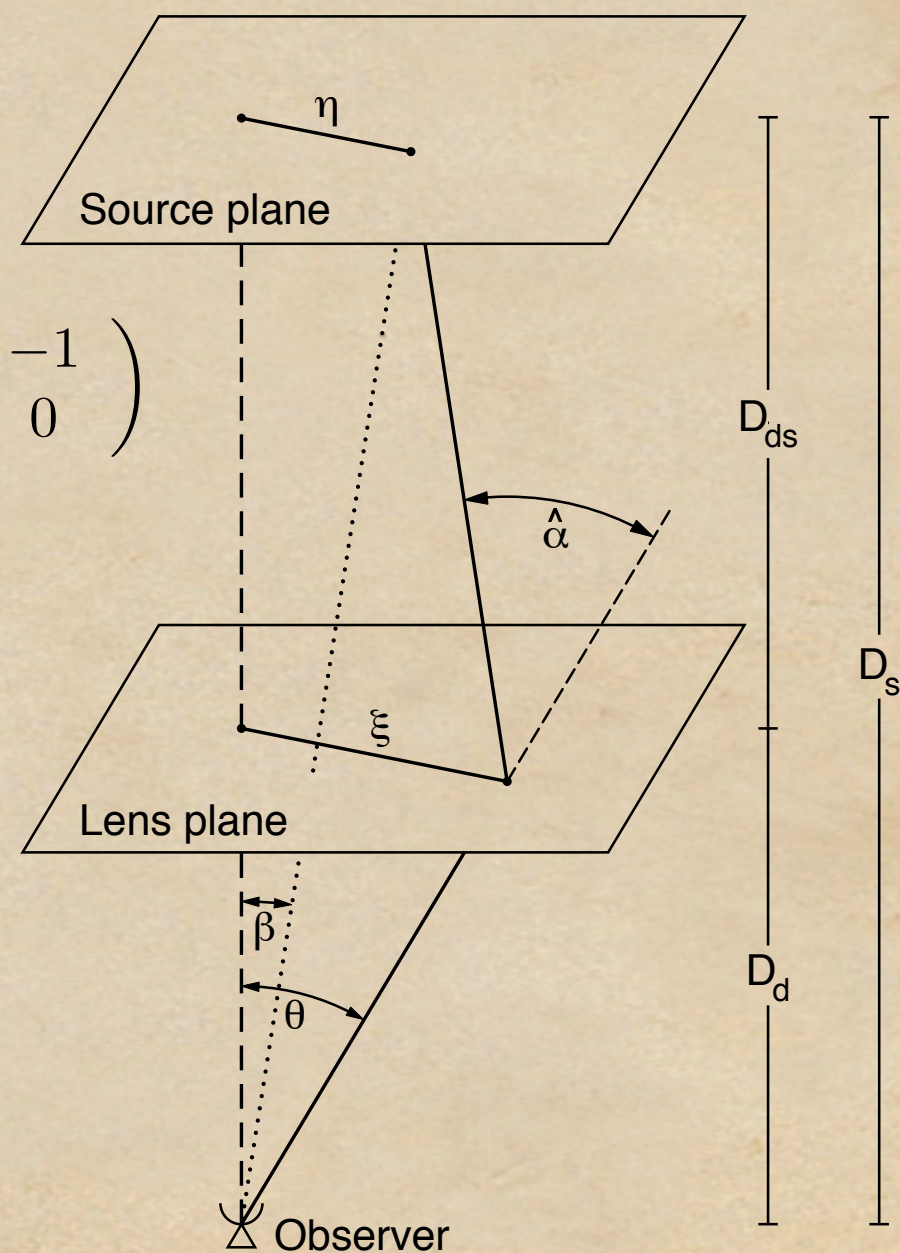
Gravitational Lensing 201

But Did we Miss Anything???

$$\begin{aligned}
 \mathcal{A}_{ij} &= \frac{\partial x_i^S}{\partial x_j^I} \\
 &= (1 - \kappa)\delta_{ij} - \gamma_{ij} + \omega\epsilon_{ij}
 \end{aligned}$$

$\downarrow$ trace
 $\downarrow$ symmetric traceless
 $\downarrow$ antisymmetric rotation

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$



Credit: Bartelmann and Schneider 2001

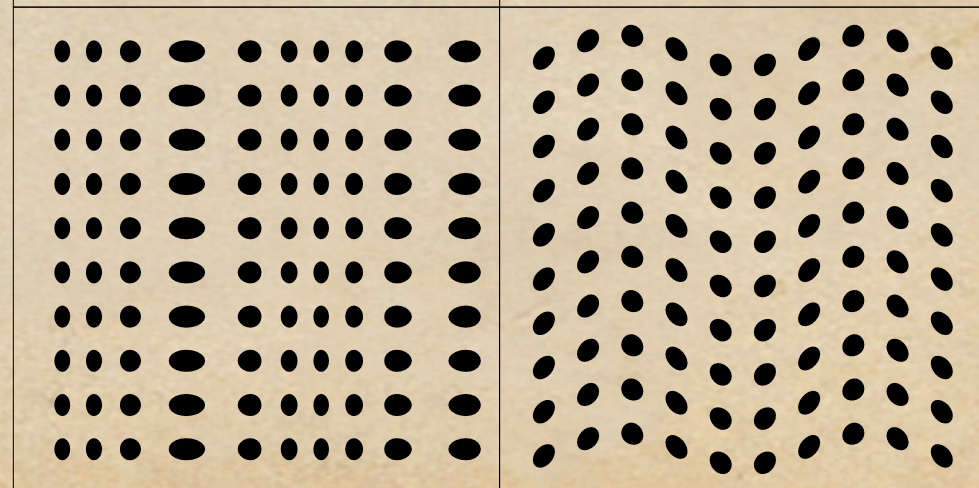
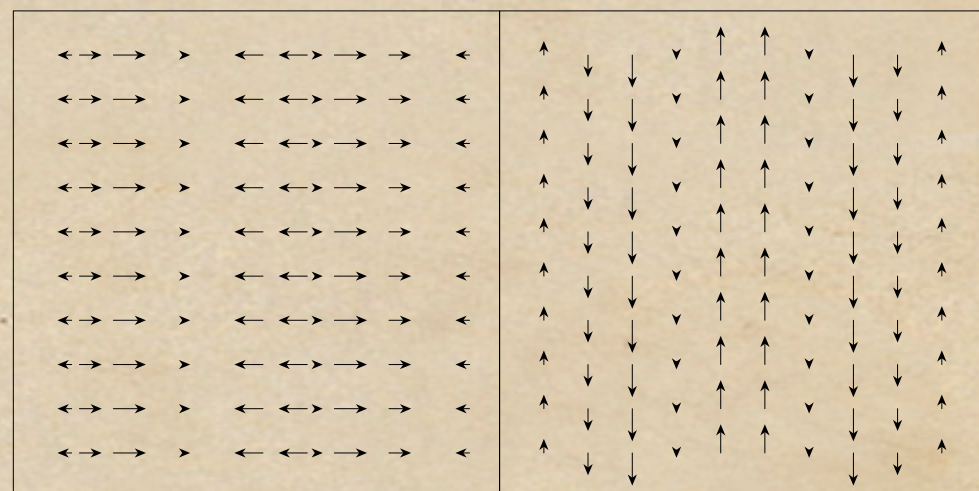
Gravitational Lensing 201

Cosmic Shear by Gravity Waves:

$$\mathcal{A} = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 - \omega \\ -\gamma_2 + \omega & 1 - \kappa + \gamma_1 \end{pmatrix}$$

Ang. deflctn proj. on the sky:

$$\vec{\Delta} = [\vec{r} - (\hat{n} \cdot \hat{r})\hat{n}] / (\eta_0 - \eta)$$



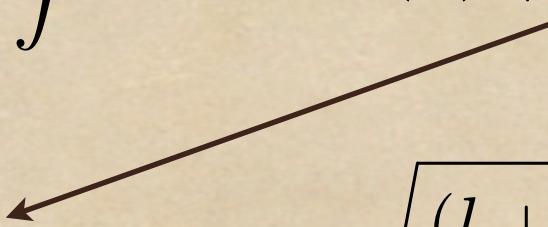
Credit: BPol, Albert Stebbins

Gravitational Lensing 201

The angular power spectrum of the rotational component:

$$\omega(\hat{n}) \equiv -\frac{1}{2}\hat{n} \cdot [\nabla \times \hat{r}(\hat{n}, \eta_S)]$$

$$\begin{aligned} C_l^{\omega\omega} &= \frac{1}{2l+1} \sum_{m=-l}^l \langle |\omega_{lm}|^2 \rangle \\ &= \frac{2}{\pi} \int k^2 dk P_t(k) |T_l^\omega(k, \eta_S)|^2 \end{aligned}$$

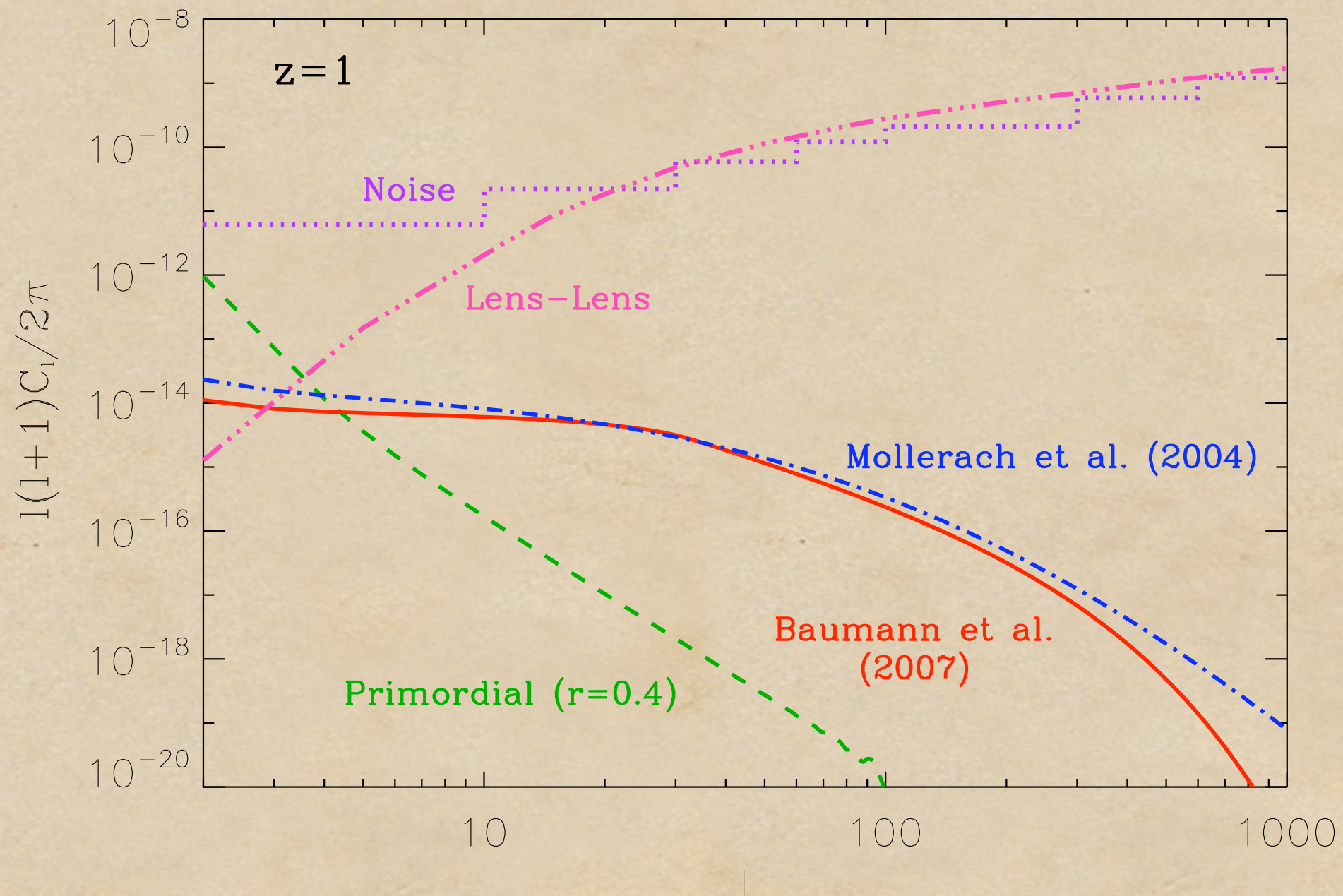


$$T_l^\omega(k, \eta_S) = \sqrt{\frac{(l+2)!}{(l-2)!}} \int_{\eta_S}^{\eta_0} k d\eta' T_t(k, \eta') \left. \frac{j_l(x)}{x^2} \right|_{x=k(\eta_0-\eta')}$$

Agenda

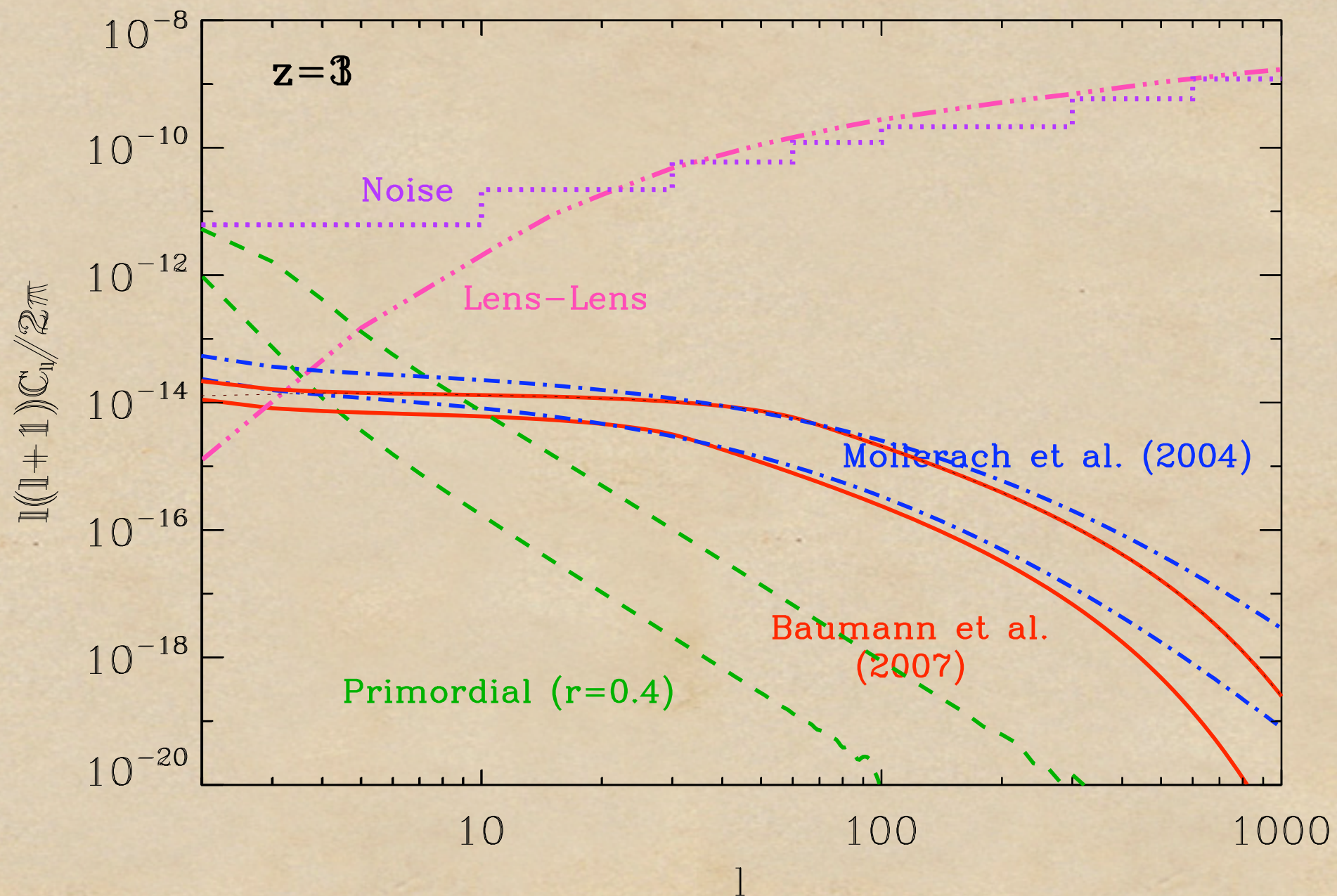
- ☑ ABC's of Gravitational Waves
- ☑ Primary GWB vs Scalar-Induced GW
- ☑ Why Cosmic Shear?
- ☑ Gravitational Lensing 201
- ☐ Method and Results
- ☐ Conclusion

Cosmic Shear Curl Mode Power Spectra



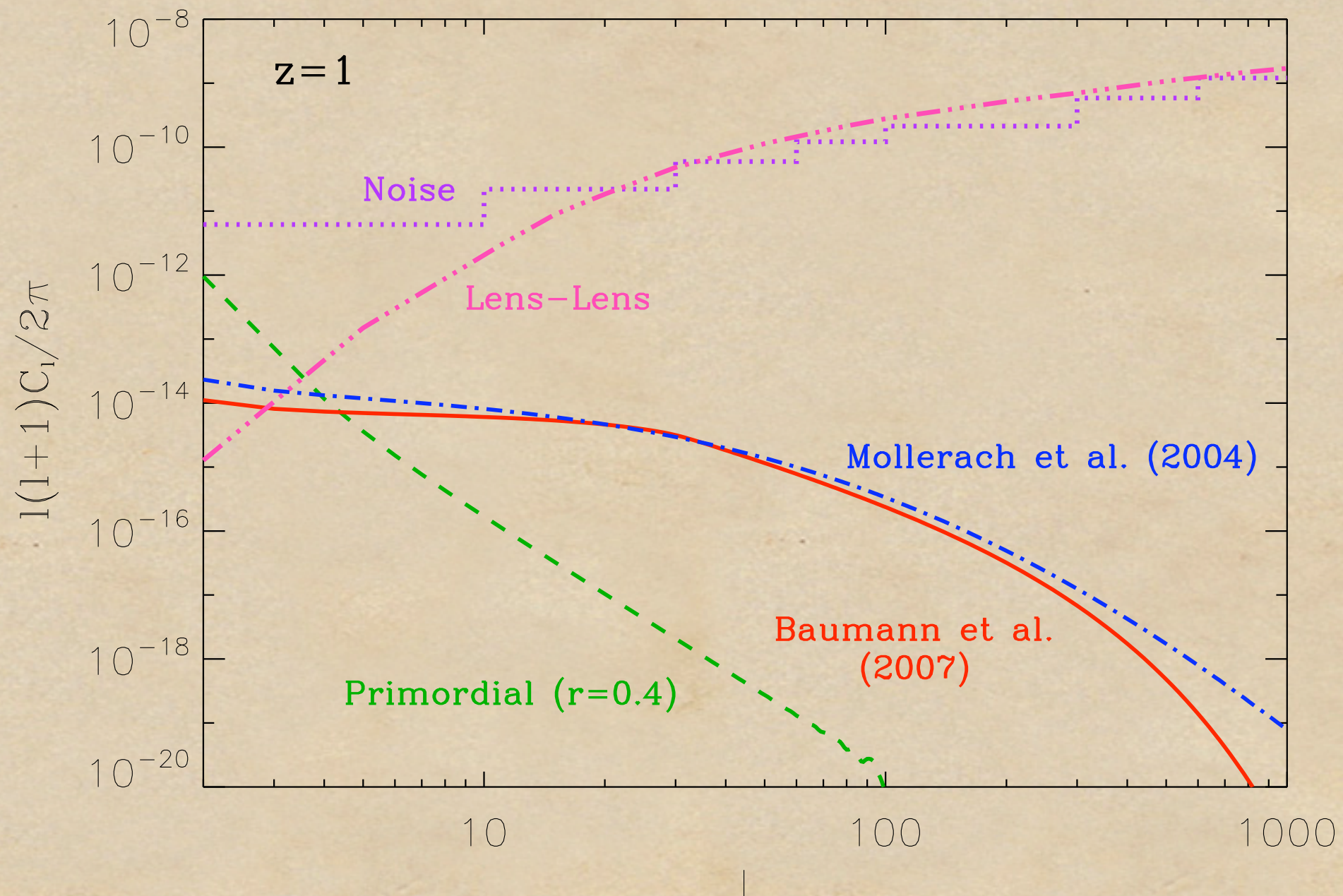
D.S., P. Serra, A. Cooray, K. Ichiki, D. Baumann, PRD, 77, 103515 (2008)

Cosmic Shear Curl Mode Power Spectra



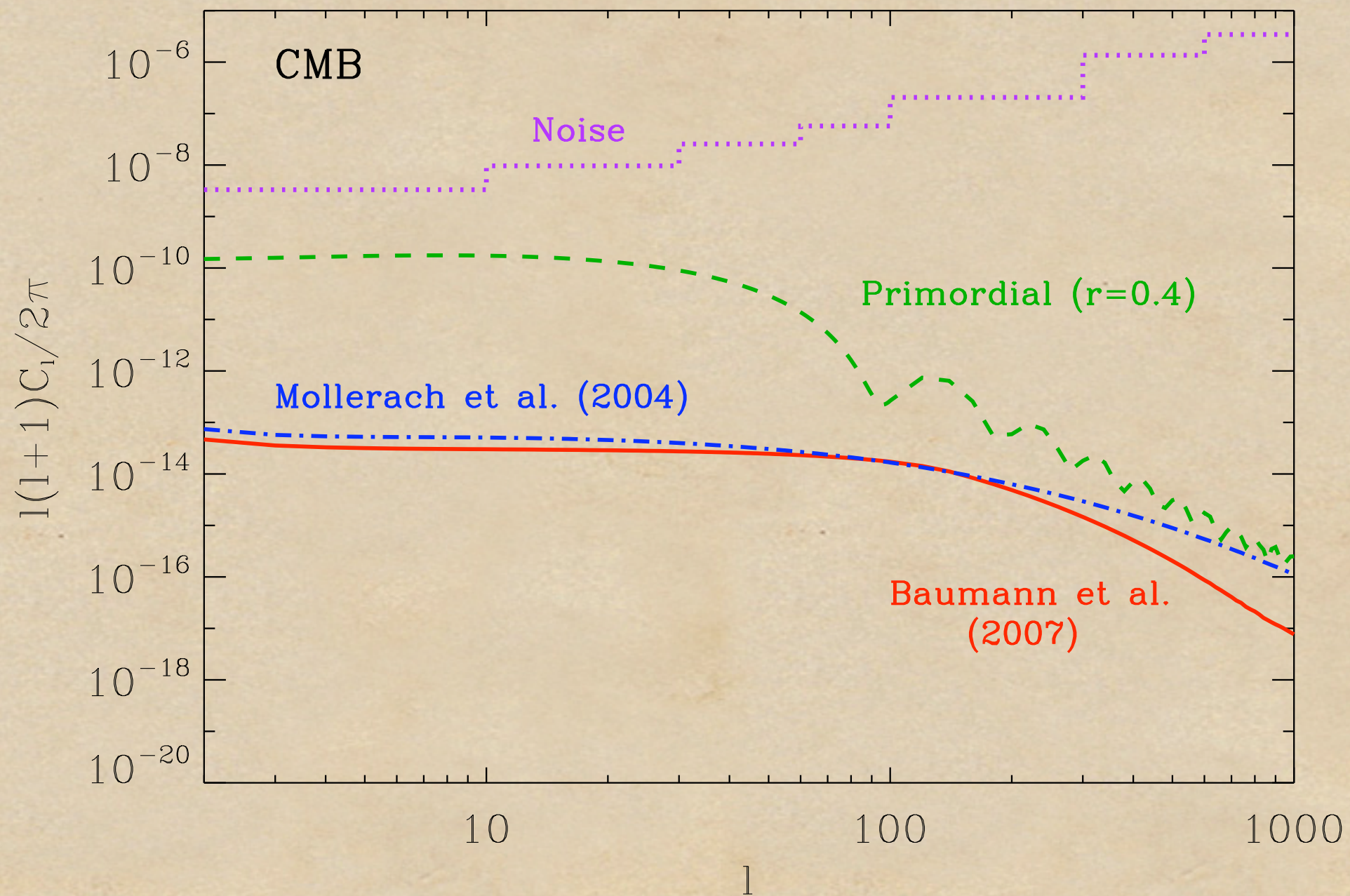
D.S., P. Serra, A. Cooray, K. Ichiki, D. Baumann, PRD, 77, 103515 (2008)

Cosmic Shear Curl Mode Power Spectra



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Lensing of CMB Anisotropies by GW



D.S., P. Serra, A. Cooray, K. Ichiki, D. Baumann, PRD, 77, 103515 (2008)